R. I. G. Hughes, The structure and interpretation of quantum mechanics, Harvard University Press, 1989, ISBN: 0674843916
http://www.directtextbook.com/prices/0674843916

(1982, pp. 64–74) did not interpret probabilities as relative frequencies, preferring instead a *propensity* interpretation.

Independently of any cachet bestowed by its pedigree, the statistical interpretation is prima facie a very plausible and attractive view of quantum theory. Unfortunately it cannot be maintained—at least, not in the simple form in which I have presented it.

6.5 *Kochen and Specker's Example*

The statistical interpretation, as presented in the previous section, will be threatened by any counterexample to PVP. Such a counterexample is offered by Kochen and Specker (1967); if their result holds, then we cannot regard the properties of systems in the way that the statistical interpretation suggests.

The example they use involves a spin-1 system. Whereas for the spin-$\frac{1}{2}$ particle there are only two possible values, $+\frac{1}{2}$ and $-\frac{1}{2}$, of any component of spin, for a spin-1 system there are three: $+1$, 0, and -1. Thus the square S^2_α of any component of spin can take as values only $+1$ and 0. Kochen and Specker show, first, that, if we take any triple of these squares, S^2_α, S^2_β, and S^2_γ, corresponding to three mutually perpendicular directions in space, α, β, and γ, then for all states of the system a measurement will show two of them to have value 1 and the third 0. PVP would then require us to assign 1 or 0 to each direction in space, and to do so in such a way that, of any three mutually perpendicular axes, α, β, γ, two receive value 1 and the third 0. By a geometrical argument, Kochen and Specker show that this cannot be done.

This is a very remarkable result—how remarkable can be seen by comparing this situation with that of the components of spin of the spin-$\frac{1}{2}$ particle, whose possible values are just $+\frac{1}{2}$ and $-\frac{1}{2}$. In this case, PVP suggests that each direction in space must receive a value different from that given to the diametrically opposed direction. Clearly, one elementary way to do this is to imagine a sphere split into two; to one hemisphere we assign $+\frac{1}{2}$, and to the other we assign $-\frac{1}{2}$. Whether or not we could ever generate the quantum-mechanical statistics from such an assignment of values is, of course, a very different question. The point is that Kochen and Specker's example shows that, for certain systems, even that trivial kind of assignment is denied us. Recall, in this connection, that Gleason's theorem applies only to a space of dimensionality three or greater. (See Section 5.6.)

6.6 *Generalizing the Problem*

The formalism of quantum mechanics entered the argument of the previous section in one place only: it was used to establish that the sum of the values of S_x^2, S_y^2, and S_z^2 must be equal to 2. However, we can use an extension of the impossibility proof to show that PVP cannot hold in any physical theory that uses the full representational capacity of a Hilbert space of three or more dimensions, that is, in which there is a one-to-one correspondence between experimental questions pertaining to a certain class of observables and the set of subspaces of such a space.

these models? If "to justify" here means more than "to show that they save the phenomena," and what is required is some deeper analysis warranting their use, then it cannot do so.

This argument is not intended to provide "a tranquilizing philosophy, . . . a gentle pillow for the true believer from which he cannot easily be aroused" (Einstein, letter to Schrödinger, May 1928, on the Copenhagen interpretation; quoted in Bub, 1974, p. 46). It is an argument which claims that the scope of quantum theory is limited by its own structure.

Landau and Lifschitz (1977, p. 3) write,

> Thus quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation.

I suggest that the explanation of how this can be so, how we can use the limiting cases of quantum theory in order to formulate the theory, cannot be given within the theory itself. It will have to await the arrival of a new physical theory, a theory which is not formulated against a classical horizon in the way that quantum mechanics is.

Can there be such a theory?

Probably.