Negative masses, even if isolated, imply self-acceleration, hence a catastrophic world

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Summary. — The conjecture of the existence of negative masses together with ordinary positive masses leads to runaway motions even if no self-reaction is considered. Pollard and Dunning-Davies have shown other constraints as a modification of the principle of least action and that negative masses can only exist at negative temperatures, and must be adiabatically separate from positive masses. We show here that the self-reaction on a single isolated negative mass implies a runaway motion. Consequently, the consideration of self-fields and relevant self-reaction excludes negative masses even if isolated.

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1. – Introduction

The conjecture of the existence of negative masses has been considered by several authors beginning with a paper by Ramsey [1], followed by an excellent review of Bondi in general relativity [2]. The previous works have been summarized in a recent paper of Pollard and Dunning-Davies [3] who introduce other constraints for the possibility of the existence of negative masses. Runaway motion for two interacting masses, one positive and one negative, is a recurrent feature throughout. Consider, for example, two equal and opposite interacting masses. Because of the equivalence principle the gravitational interaction is repulsive. However, because of Newton's law, the negative mass accelerates toward the positive mass and a runaway motion starts. In special relativity (SR) the velocities of both particles tend to the speed of light c.

Apart from some works of science fiction [4], other drawbacks have been found beside the runaway motion. For instance, Ramsey [1] has shown that a gas of negative-mass molecules implies negative temperatures and, consequently, that the Kelvin statement of the second principle of thermodynamics has to be altered in order to incorporate negative temperatures.

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Bonner [5] has pointed out that a negative-mass gas supports tensions rather than pressures in spite of the collisions and bouncings of the molecules on the walls.

Pollard and Dunning-Davies [3] introduce a necessary modification to the principle of least action to account for negative masses. Moreover, they give other reasons to the conclusion of previous authors [1, 2, 5] that positive and negative masses cannot coexist. They emphasize that negative masses can only exist at negative temperatures, and must be adiabatically separated from positive masses. This separation, according to them, would rule out runaway motion. Our conclusion is that even this separation is unable of preventing the runaway motion. Actually, as shown in sect. **2**, the self-reaction of any kind of field on itself brings about a runaway motion of an isolated negative mass.

To our knowledge there is only Winterberg [6] who has introduced negative masses together with positive masses at the same time disregarding the runaway motions. Precisely, Winterberg, in his main paper [6], has proposed that there might be an underlying non-relativistic superfluid substratum of densely packed positive and negative Planck masses, permeating all of space, and making up what may be called the Planck ether. Winterberg, in his alleged "Planck Aether Theory", assumes positive Planckions (with inertial and gravitational masses both positive) and negative Planckions (with inertial and gravitational masses both negative), so that for both types of Planckions Einstein's equivalence principle is formally satisfied. The introduction of negative masses has two aims, namely: i) to have a cosmological constant (in general relativity) exactly equal to zero; ii) to overcome the divergence problems of relativistic quantum field theories.

Winterberg does not discuss the possibility, or not, to eliminate the runaway motions. Perhaps he thinks to have avoided them by assuming point-contact forces between his Planckions. Actually, point-contact forces are very short-range interactions. In a head-on collision between two opposite Planckions the positive one bounces normally while the negative Planckion bounces toward the positive one so that the contact collision goes on for ever implying a continuous acceleration of the Planckion pair. To avoid this kind of runaway motion the positive Planckion must not interact with the negative Planckion. This is equivalent to the separation of positive masses from negative masses as the unique condition of existence of negative inertia.

However, if we show that a single, isolated, negative Planckion brings about a self-acceleration the assumption of absence of interaction between positive and negative Planckions is of no help.

The self-acceleration is due to the fields internal to each Planckion, fields that are necessary to keep together each Planckion and to produce the stresses consequent to collisions. In passing, we notice that if the Planckions have only short-range interactions (as claimed by Winterberg), it is impossible to explain the origin of the long-range electromagnetic and gravitational interactions as attempted by Winterberg [6].

2. – The assumption of a negative mass implies its self-acceleration

We show that a negative mass, with negative total energy, has a negative inertia so that it auto-accelerates and the kinetic energy would tend to minus infinity. The best way to show that a particle with negative inertia undergoes a self-acceleration is to consider the contribution to the inertia due to the acceleration field radiated by the particle itself. During all its past history the particle has undergone at least one action by part of an external field (the action is continuous if we consider the vacuum fluctuations due to the zero-point field). When the particle was accelerated any part of it radiated an acceleration field which acted on all the other parts of the particle at a retarded time. The mutual actions of all the parts of the particle produce a force opposite to the acceleration. If the mass is positive (as occurs for all the physical particles) the retarded force brings about an acceleration smaller than, and opposite to, the initial acceleration. But if the inertia is negative the initial acceleration is increased and an avalanche process occurs. Let us formalize what said by using the electromagnetic field for simplicity. For the gravitational field the self-reaction has been explicitly calculated [7] and the result is similar for any interaction as shown in field theory [8], where energy conservation is valid (obviously, for positive masses). The relativistic equation of motion for a particle of bare mass m_0, charge q, volume V is

$$m_0 \frac{\mathrm{d}(\gamma \mathbf{v})}{\mathrm{d}t} = q\left(\mathbf{E}_{\mathrm{ext}} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{\mathrm{ext}}\right) + \iiint_V \mathrm{d}^3 r\, \varrho \left(\mathbf{E}_{\mathrm{s}} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{\mathrm{s}}\right), \tag{1}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, the subscript “ext” stands for “external”, and the subscript “s” for self-fields given by the Lienard-Wiechert solutions:

$$\begin{cases} \mathbf{E}_{\mathrm{s}}(\mathbf{r}, t) = \displaystyle\iiint_V \mathrm{d}^3 r'\, \frac{\varrho(\mathbf{r}', t_{\mathrm{a}})}{(1 - \hat{\mathbf{r}}\cdot\mathbf{v}/c)^3} \left\{ \frac{\hat{\mathbf{r}} - \mathbf{v}/c}{|\mathbf{r} - \mathbf{r}'|^2}\left(1 - \frac{v^2}{c^2}\right) + \frac{[\mathbf{a} \times (\hat{\mathbf{r}} - \mathbf{v}/c)] \times \hat{\mathbf{r}}}{c^2 |\mathbf{r} - \mathbf{r}'|} \right\}, \\ \mathbf{B}_{\mathrm{s}}(\mathbf{r}, t) = \displaystyle\iiint_V \mathrm{d}^3 r'\, \frac{\varrho(\mathbf{r}', t_{\mathrm{a}})}{(1 - \hat{\mathbf{r}}\cdot\mathbf{v}/c)^3} \cdot \\ \qquad \cdot \left\{ \dfrac{(\mathbf{v}/c) \times \hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|^2}\left(1 - \dfrac{v^2}{c^2}\right) + \dfrac{\mathbf{a}\cdot\hat{\mathbf{r}}(\mathbf{v}/c)] \times \hat{\mathbf{r}} + (1 - \hat{\mathbf{r}}v/c)\, \mathbf{a} \times \hat{\mathbf{r}}}{c^2 |\mathbf{r} - \mathbf{r}'|} \right\}, \end{cases} \tag{2}$$

where the non-rationalized Gauss system is used, $\mathbf{a}(t) = \mathrm{d}\mathbf{v}/\mathrm{d}t$, and

$$\hat{\mathbf{r}} = (\mathbf{r} - \mathbf{r}')/|\mathbf{r} - \mathbf{r}'| \quad \text{and} \quad t_{\mathrm{a}} = t - |\mathbf{r} - \mathbf{r}'|/c\,. \tag{3}$$

To show the self-acceleration of a charged negative mass it is sufficient to consider incipient motion for which we can neglect v/c so that $\gamma \longrightarrow 1$, and eq. (1) reduces to

$$m_0 \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = q\mathbf{E}_{\mathrm{ext}} + \iiint_V \mathrm{d}^3 r\, \varrho \mathbf{E}_{\mathrm{s}}\,, \tag{4}$$

with

$$\mathbf{E}_{\mathrm{s}}(\mathbf{r}, t) \simeq \iiint_V \mathrm{d}^3 r'\, \varrho(\mathbf{r}', t_{\mathrm{a}}) \left\{ \frac{\hat{\mathbf{r}}}{|\mathbf{r} - \mathbf{r}'|^2} + \frac{(\mathbf{a} \times \hat{\mathbf{r}}) \times \hat{\mathbf{r}}}{c^2 |\mathbf{r} - \mathbf{r}'|} \right\}. \tag{5}$$

Let us substitute eq. (5) into eq. (4) when the external fields have vanished. The first term inside the curly bracket of eq. (5) represents the velocity field and its resultant action is zero. The second term is the acceleration field and its resultant is different from zero as can immediately be seen from the double cross product. Both $\mathbf{v}$ and $\mathbf{a} = \mathrm{d}\mathbf{v}/\mathrm{d}t$ in the integral are calculated at the advanced time t_a (with respect to the present time t of the field point). Consequently, eq. (4) becomes, if all the parts of the particle have the same acceleration at time t, *i.e.* if $\mathbf{a}$ depends on $\mathbf{r}'$ only through $t_a(\mathbf{r}, \mathbf{r}')$,

$$m_0\,\mathbf{a}(t) = -\iiint_V \mathrm{d}^3 r\,\varrho(\mathbf{r}, t)\iiint_V \mathrm{d}^3 r'\;\varrho(\mathbf{r}', t_a)\;\frac{\mathbf{a}(t_a) - \mathbf{a}(t_a)\cdot\hat{\mathbf{r}}\hat{\mathbf{r}}}{c^2\,|\mathbf{r}-\mathbf{r}'|}\;. \tag{6}$$

We apply eq. (6) to the case of a sphere of radius R having a charge density $\varrho(\mathbf{r}, t)$ with spherical symmetry. Presumed that $\mathbf{a}$ kept the same direction at least from $t - 2R/c$ to t, projecting eq. (6) on this direction, and calling ξ the angle between $\mathbf{a}$ and $\hat{\mathbf{r}}$, gives

$$m_0 a(t) = -\iiint_V \mathrm{d}^3 r\,\varrho(\mathbf{r}, t)\iiint_V \mathrm{d}^3 r'\;\varrho(\mathbf{r}', t_a)\;a(t_a)\;\frac{1-\cos^2\xi(\mathbf{r}, \mathbf{r}')}{c^2\,|\mathbf{r}-\mathbf{r}'|}\;. \tag{7}$$

The whole integrand is positive so that the right-hand side (RHS) of eq. (7) is negative. Consequently, if $m_0 > 0$ the RHS of eq. (7) is < 0, while the LHS is > 0. There are no possibilities to find a positive-definite function $a(t)$ that satisfies the integral equation (7) and *physical* runaway solutions are excluded: the only physical solution of the integral equation (7) is $\mathbf{a}(t) = 0$. This result can also be obtained by performing a Lagrange expansion of $a(t_a)$ around $t_a = t$, thus obtaining the Lorentz series for the RHS of eq. (7). It has been proved [9] that the equation of motion with the Lorentz series is equivalent to the following finite-difference equation:

$$\mathbf{F} = \frac{m_0}{\Delta t}\,[\mathbf{v}(t) - \mathbf{v}(t-\Delta t)]\,, \tag{8}$$

where $\Delta t = 1.5R/c$ is the time taken by light to cross 1.5 times the particle radius R (typically the Lorentz electron radius $R_L = e^2/(m_0 c^2)$). When $\mathbf{F} = 0$, then eq. (8) becomes $\mathbf{v}(t) = \mathbf{v}(t-\Delta t)$ whose only solution is $\mathbf{v}(t) = \mathrm{const}$, hence $\mathbf{a} = 0$. The famous *spurious* runaway solutions arise if one truncates the series after two terms (the second term is the well-known radiation damping $2e^2(3c^3)^{-1}\,\mathrm{d}a/\mathrm{d}t$, where e is the total charge). The relevant runaways are unphysical and a consequence of the series truncation.

On the contrary, if $m_0 < 0$ it is possible to find a function $a(t_a)$ in the time interval from $t - 2R/c$ to t such that both sides of eq. (7) become equal. If we take $a(t_a) =$ constant, the RHS of eq. (7) becomes $m_{\mathrm{e.m.}}a$, where $m_{\mathrm{e.m.}}$ is the electromagnetic mass due to the self-reaction which, if ϱ is constant as well, turns out to be given by $m_{\mathrm{e.m.}} = 4e^2/(5R)$ which is equal to c^{-2} times the sum of the electrostatic and the Poincaré energy:

$$m_{\mathrm{e.m.}} = \frac{4e^2}{5Rc^2} = \frac{1}{c^2}\,(U_{\mathrm{e.s.}} + U_{\mathrm{Poincaré}}) = \frac{3e^2}{5Rc^2}\left(1 + \frac{1}{3}\right). \tag{9}$$

The Poincaré stresses (and the relevant energy) are necessary to prevent the charge to explode because of the repulsive action of its own charge. We do not therefore agree with Rohrlich [10] who disregarded the Poincaré stresses and changed the usual expression of the electromagnetic momentum. The e.m. mass $m_{\text{e.m.}}$ is due to the self-action and is independent of any definition of the e.m. momentum. Moreover, eq. (9) shows the consistency of $m_{\text{e.m.}} c^2$ with the *total* energy (including the Poincaré one).

If $m_{\text{e.m.}} > |m_0|$, then $a(t_a)$ has to be smaller than $a(t)$. A typical function satisfying this condition is an increasing exponential. If $m_{\text{e.m.}} < |m_0|$, then $a(t_a)$ must be larger than $a(t)$ that turns out to be given by a decreasing exponential. In any case there is a spontaneous self-acceleration. In the first case, the velocity increases exponentially while in the second case it reaches an asymptotic value.

In the second case (*i.e.* when $m_{\text{e.m.}} < |m_0|$) a world with negative masses is not so catastrophic as in the case $m_{\text{e.m.}} > |m_0|$ (where an exponential increase of the acceleration would occur). However, we have to take into account the presence of the ubiquitous zero-point field (ZPF), which causes fluctuations for any charged particle. A charged particle with $m_0 < 0$ and $m_{\text{e.m.}} < |m_0|$ would have a good probability to start an amplified variation of velocity at any fluctuation caused by the ZPF. The observable effect would be by far due to the consequent diffusion of the particle positions rather than to the amplified velocity fluctuations. The direct diffusion caused by the ZPF on an electron (that has a positive mass as that of all the observed particles) is negligible if the particle is free in space. Its component along the x-axis turns out to be given by [11]

$$(10) \qquad \Delta x^2 = \langle [x(t_0 + t) - x(t_0)]^2 \rangle =$$

$$= \frac{\hbar\tau}{\pi m}\left[1.1544 + 2\,\ln\,\frac{t}{\tau} - \exp\left[-\frac{t}{\tau}\right] E_{\mathrm{i}}\left[\frac{t}{\tau}\right] - \exp\left[\frac{t}{\tau}\right] E_{\mathrm{i}}\left(-\frac{t}{\tau}\right)\right],$$

where E_{i} is the exponential integral, and $\tau = 2e^2/(3mc^3)$ is two-thirds the time taken by light to cross the classical electron radius. For $t \gg \tau/\alpha$, where $\alpha = e^2/(\hbar c)$ is the fine-structure constant, eq. (10) reduces to

$$(11) \qquad \Delta x^2 \simeq \frac{\hbar\tau}{\pi m}\left(1.1544 + 2\,\ln\,\frac{2\alpha t}{3\tau}\right).$$

This expression applied to an electron in a TV set gives $\Delta x^2 \simeq 13 R_{\mathrm{C}} R_{\mathrm{L}}$, where R_{C} is the Compton radius and R_{L} the classical electron radius, so that $\sqrt{\Delta x^2} \simeq 10^{-11}$ cm, which is absolutely negligible.

This very low diffusion is due to the highly non-Markovian character of the ZPF stochastic process. If some damped runaways (present in the hypothetical case $m_0 < 0$ and $m_{\text{e.m.}} < |m_0|$) are triggered by the ZPF fluctuations, the stochastic process becomes Markovian with a diffusion given by

$$(12) \qquad \Delta x^2 = 4Dt\,,$$

where $D \simeq \lambda\,\Delta v/3$ is the diffusion coefficient, λ the mean free path, and Δv the velocity variation due to the self-acceleration. Even with a low Δv, for instance $\Delta v \simeq 10^6$ cm/s, and $\lambda \simeq 1$ cm, in a time $t \simeq 10^{-8}$ s (as that taken by an electron to travel from the cathod to the screen of a TV set) it would be $\Delta x^2 \simeq 10^{-2}$ cm^2 or $\sqrt{\Delta x^2} \simeq 1$ mm, much higher than what observed.

3. – Conclusions

From time to time many authors have considered the possibility of existence of particles with negative masses. Some authors [1, 2] have examined this possibility by an excellent state-of-the-art, pointing out important limitations to the hypothetical existence of negative masses. The most severe criticism against the simultaneous presence of both positive and negative masses is the consequent appearance of runaway motions. This is a glutton morsel for science fiction and, in fact, Forward [4] hypothetized space ships composed of positive and negative masses.

A completely different, and isolated, attitude is that of Winterberg [6] who introduced a substratum, or ether, made of equal and opposite particles he denoted as Planckions. The "opposite" includes negative masses as well, and this is the worst condition since runaways occur. Winterberg seems to be completely unaware of this fact and he does not even mention it, at least to suggest a hint for avoiding them. The reason why he introduced a non-relativistic substratum of densely packed positive and negative Planck masses, was to have a cosmological constant exactly equal to zero. As is known, the gravest problem in relativistic cosmology is the presence of quantum vacuum with fluctuation fields, the most known of which is the zero-point field (ZPF). The relevant specific energy (per unit volume) depends on the truncation ω_{tr} of the power spectral density of the ZPF. Even if we take the minimum value for ω_{tr}, *i.e.* the Compton, or spin, angular frequency ω_c, the ZPF energy density would be so high that the Universe would be closed in few meters, according to general relativity. There are two possibilities to overcome this drawback. The first one is that proposed by Winterberg but, in our opinion, his remedy is worse than the original evil, since runaways occur. The second possibility is that proposed by us [12], *i.e.* to change the gravitational theory taking the gravitational field proportional to the spatial gradient of ZPF.

The most recent paper on the subject of negative masses is that of Pollard and Dunning-Davies [3] who confirmed and clarified other queer aspects emphasized by previous authors, as the negative temperatures implied by a gas of negative-mass particles [1], and that a negative-mass gas supports tensions [5] rather than pressures (in spite of the collisions of the molecules on the walls). Pollard and Dunning- Davies [3] also point out that a modification has to be made to the principle of least action to account for negative masses. They conclude that negative masses can only exist at negative temperature, and must be adiabatically separated from positive masses.

Not even this last condition is sufficient to prevent the existence of runaways if self-reaction is taken into account as done in sect. **2**. Our conclusion is therefore the final step of the previous restrictions, thus excluding the possibility of existence of negative masses (if, as usual, the masses are defined over a field with positive units. For an exception, see Santilli [13]). In particular, Winterberg's Planckions are excluded even if the fields responsible for their mutual actions have an extremely short range. In fact the self-reaction due to the internals fields (necessary to keep together the different parts of a Planckion that cannot be point-like otherwise it would be collisionless and therefore as non-existent) brings about a self-acceleration. Moreover, the Planckions must possess electric charges, otherwise it would be impossible to explain the arising of electric charges, hence of long-range interactions, if the Planckions possess no electric charge.

Concluding, self-reaction leads to the final virdict against the possibility of existence of negative masses that would lead to a catastrophic world.

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