Ioannis Raptis, Presheaves, Sheaves and their Topoi in Quantum Gravity and Quantum Logic; paper of a talk given at the 5th Biannual International Quantum Structures Association Conference in Cesena, Italy (March-April 2001), arXiv:gr-qc/0110064v1, Sec. 2:

At the same time, and from a physical point of view, the unreasonableness and unphysicality of the locally Euclidean topological ($\mathcal{C}^0$) and differential ($\mathcal{C}^\infty$) manifold model M for spacetime is especially pronounced when one considers:

- (a) **Pointedness of events:** M's pathological nature in the guise of singularities that plague general relativity—the classical theory of gravity—which are mainly due to the geometric point-like character of the events that constitute it, as well as due to the algebras of $\mathcal{C}^\infty$-smooth functions employed to coordinatize these point events [48] (and also due to (b) next).
- (b) **Continuous infinity of events:** M's problematic nature due to the fact that one can in principle pack an uncountable infinity of the aforementioned point events in a finite spacetime volume resulting in the non-renormalizable infinities that impede any serious attempt at uniting quantum mechanics with general relativity (at least at the 'calculational' level)[4].
- (c) **Non-dynamical and non-quantal topology:** Its non-variable and non-quantal nature when one expects that at Planck scales not only the spacetime metric, but also that the spacetime topology partakes into quantum phenomena [80], that is to say, it is a dynamically variable entity whose connections engage into coherent quantum superpositions. We may distill this by saying that the manifold topology is, quantally speaking, an *unobservable* entity not manifesting quantum dynamical fluctuations or interference between its defining connections [62, 58]—a rigid substance, once and forever fixed by the theorist, that is not part of the dynamical flux of Nature at microscopic scales.

[2] That is, in general relativity at least, the gravitational field, which is represented by the spacetime metric $g_{\mu\nu}$, is treated as an *observable*; in fact, the sole spacetime observable.

[3] That is to say, not all mathematicians and mathematical physicists agree on what ought to qualify as '*noncommutative topology*' proper and its related noncommutative sheaf or scheme theory. At the same time, there is no collective agreement on how such a noncommutative or quantum [34, 35, 36, 30, 29] topology may be applied to the problem of the quantum structure and dynamics of spacetime.

[4] The (a) and (b) pathological features of the manifold model above may be summarized in its character as *a geometric point set differential continuum of events* [48].

Furthermore, the (algebras of) commutative $\mathcal{C}^\infty$-determinations of the manifold's point events indicate another non-quantal (classical) feature of the spacetime manifold [61, 48].

- (d) **Additional structures:** M's need of extra structures required to be introduced by hand by the theoretician and not being 'naturally' related to the topological manifold (*ie*, the $\mathcal{C}^0$-continuous) one. Such structures are the differential (*ie*, the $\mathcal{C}^\infty$-smooth) and Lorentzian metric (*ie*, the smooth metric field $g_{\mu\nu}$ of absolute signature 2) ones [5], and they are implicitly postulated by the general relativist on top of M's fixed continuous topology in order to support the apparently necessary full differential geometric (*ie*, Calculus based!) panoply of general relativity. The 4-dimensional, $\mathcal{C}^\infty$-smooth Lorentzian manifold assumption for spacetime concisely summarizes the kinematics of general relativity [73, 47, 62, 48].