Mach's Principle

J V Narlikar

Jayant V Narlikar is a cosmologist and theoretical astrophysicist. He was a research student and a long-time collaborator of Fred Hoyle. He is the Founder Director of IUCCA and is currently an emeritus professor there. He has made strong efforts to promote teaching and research in astronomy in the universities. He has written extensively in English and Marathi to popularize science.

The underlying idea in Mach's principle is that the origin of inertia or mass of a particle is a dynamical quantity determined by the environment, in particular the rest of the matter in the universe. In this article, we discuss the role of this idea in the Brans–Dicke theory of gravitation and the Hoyle–Narlikar cosmology.

1. Absolute Space Revisited

There are two ways of measuring the Earth's spin about its polar axis. By observing the rising and setting of stars, the astronomer can determine the period of one revolution of the Earth around its axis: the period of $23^{h}56^{m}4^{s}$. The second method employs a Foucault pendulum whose plane gradually rotates around a vertical axis as the pendulum swings (see *Figure* 1). Knowing the latitude of the location of the pendulum it is possible to calculate the Earth's spin period. The two methods give the same answer.

At first sight this does not seem surprising. If we are measuring the same quantity, we should get the same answer regardless of the method used. Closer examination, however, reveals why the issue is nontrivial. The two methods are *not measuring the same quantity.* The first method measures the Earth's spin period against a background of distant stars, while the second employs the standard Newtonian mechanics in a spinning frame of reference. In the latter case, we take note of how laws of motion get modified when their consequences are measured in a frame of reference spinning relative to the 'absolute space' in which these laws were first stated by Newton.

Keywords
Inertia, Newton's bucket experiment, cosmology.

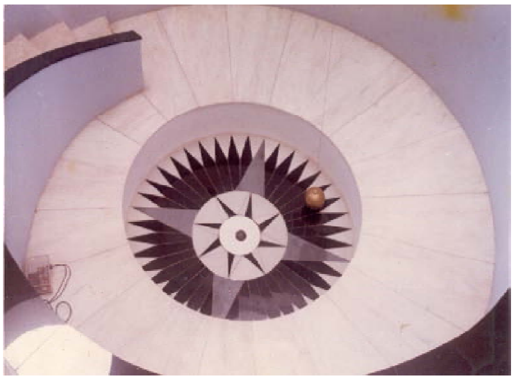


Figure 1. A working model of Foucault's pendulum at IUCAA, Pune. The vertical plane of oscillation of the pendulum takes approximately 75 hours to make one round.

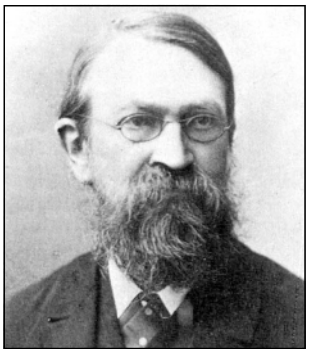
Thus, implicit in the assumption that equates the two methods is the coincidence of absolute space with the background of distant stars. It was Ernst Mach (see *Figure* 2) in the late nineteenth century who pointed out that this coincidence is nontrivial. He read something deeper in it, arguing that the postulate of absolute space that allows one to write down the laws of motion and arrive at the concept of inertia is somehow intimately related to the background of distant parts of the universe. This argument is known as 'Mach's principle' and we will analyse its implications further.

When expressed in the framework of the absolute space, Newton's second law of motion takes the familar form

$$\mathbf{P} = m\mathbf{f}. \tag{1}$$

This law states that a body of mass m subjected to an external force $\mathbf{P}$ experiences an acceleration $\mathbf{f}$. Let us denote by Σ the coordinate system in which $\mathbf{P}$ and $\mathbf{f}$ are measured.

Figure 2. Ernst Mach (1838–1916).



Newton was well aware that his second law has the simple form (1) only with respect to Σ and those frames that are in uniform motion relative to Σ. If we choose another frame Σ' that has an acceleration $\mathbf{a}$ relative to Σ, the second law of motion measured in Σ' becomes

$$\mathbf{P}' \equiv \mathbf{P} - m\mathbf{a} = m\mathbf{f}'. \tag{2}$$

The additional force is proportional to the inertial mass of the body. Newton discusses this force at length in his *Principia*, citing the example of a rotating water-filled bucket.

Although (2) outwardly looks the same as (1), with $\mathbf{f}'$ the acceleration of the body in Σ', something new has entered into the force term. This is the term $m\mathbf{a}$, which has nothing to do with the external force but depends solely on the mass m of the body and the acceleration $\mathbf{a}$ of the reference frame relative to the absolute space. Realizing this aspect of the additional force in (2), Newton termed it "inertial force". As this name implies, the additional force is proportional to the inertial mass of the body. Newton discusses this force at length in his *Principia*, citing the example of a rotating water-filled bucket. Let us look at that experiment.

2. Newton's Bucket Experiment

In *Figure* 3a we have shown a bucket full of water hanging by a rope tied to the ceiling. Suppose the rope is given a twist and let go. In *Figure* 3b we see the same bucket turning around and around as a result of the rope unwinding. To an observer in the room the bucket appears spinning in state (b). However, a fly sitting on the bucket might conclude that in the bucket is at rest and the room is spinning. The same fly would have concluded that in (a) the room was not spinning. However, note that there is one difference between states (a) and (b). The water surface in (a) is flat and horizontal, while that in (b) is curved inwards, rising at the rim. Why this curvature? The fly would reason that the curvature of the water surface is due to the centrifugal force that acts

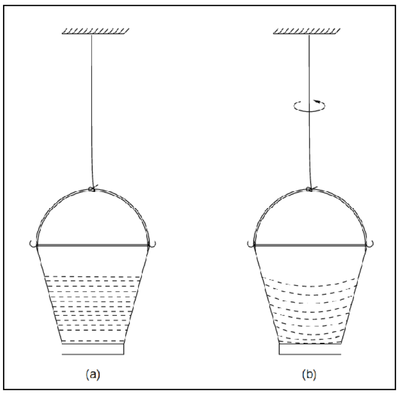


Figure 3. Newton's bucket experiment (see text for details).

on the water mass. Being radially outwards, it would produce the observed effect on the water. This example was discussed by Newton in his *Principia*. Newton argued that in (a) the bucket is at rest relative to the absolute space, while in (b) it is rotating relative to the absolute space and hence extra inertial forces have to be postulated to explain the curvature of the water surface. The centrifugal force is the inertial force in this example.

According to Mach, the Newtonian discussion was incomplete in the sense that the existence of the absolute space was postulated arbitrarily and in an abstract manner. Why does Σ have a special status in that it does not require the inertial force? How can one physically identify Σ without recourse to the second law of motion, which is based on it?

According to Mach, the Newtonian discussion was incomplete in the sense that the existence of the absolute space was postulated arbitrarily and in an abstract manner.

To Mach the answers to these questions were contained in the observation of the distant parts of the universe. It is the universe that provides a background reference frame that can be identified with Newton's frame Σ. Instead of saying that it is an accident that Earth's rotation velocity relative to Σ agrees with that relative to the distant parts of the universe, Mach took it as proof that the distant parts of the universe somehow enter into the formulation of local laws of mechanics.

Instead of saying that it is an accident that Earth's rotation velocity relative to Σ agrees with that relative to the distant parts of the universe, Mach took it as proof that the distant parts of the universe somehow enter into the formulation of local laws of mechanics.

3. Inertia in an Empty Universe

One way this could happen is by a direct connection between the property of inertia and the existence of the universal background. To see this point of view, imagine a single body in an otherwise empty universe. In the absence of any forces, (1) becomes

$$m\mathbf{f} = \mathbf{0}. \tag{3}$$

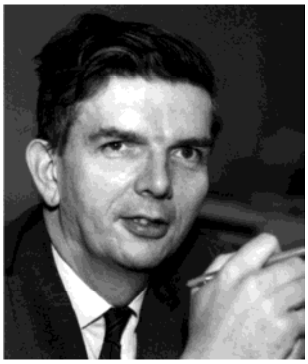
What does this equation imply? Following Newton we would conclude from (3) that $\mathbf{f} = \mathbf{0}$, that is, the body moves with uniform velocity. But we now no longer have a background against which to measure velocities. Thus

$\mathbf{f} = \mathbf{0}$ has no operational significance. Rather, the lack of any tangible background for measuring motion suggests that $\mathbf{f}$ should be completely indeterminate. And it is not difficult to see that such a conclusion follows naturally, provided we come to the remarkable conclusion, also possible from (3) that

$$m = 0. \tag{4}$$

In other words, the measure of inertia depends on the existence of the background in such a way that in the absence of the background the measure vanishes! This aspect introduces a new feature into mechanics not considered by Newton. The Newtonian view that inertia is the property of matter has to be augmented to the statement that inertia is the property of matter as well as of the background provided by the rest of the universe. This general idea is a consequence of Mach's principle.

Figure 4.
***Top: Carl Brans* (1935–)**
Bottom: Robert H Dicke
(1916–1997).



Such a Machian viewpoint not only modifies local mechanics, but it also introduces new elements into cosmology. For, there is no basis now for assuming that particle masses would necessarily stay fixed in an evolving universe. This is the reason for considering cosmological models anew from the Machian viewpoint. Although Mach himself never gave a quantitative expression to these ideas, a few other scientists have done so. Presented here are some instances of how different physicists have given quantitative expression to Mach's principle and arrived at new cosmological models.

4. The Brans–Dicke Theory of Gravity

In 1961, C Brans and R H Dicke (see *Figure* 4) provided an interesting alternative to general relativity based on Mach's principle. To understand the reasons leading to their field equations, we first note that the concept of a variable inertial mass arrived at in Section 3 itself leads to a problem of interpretation. For, how do we compare masses at two different points in spacetime?

Masses are measured in certain units, such as masses of elementary particles, which might themselves be subject to change! We need an independent unit of mass against which an increase or decrease of a particle mass can be measured. Such a unit is provided by gravity, by the so-called Planck mass defined by $m_{\rm P}$:

$$m_{\rm P} \equiv \left(\frac{\hbar c}{G}\right)^{1/2} \cong 2.16 \times 10^{-5}\text{g}. \qquad (5)$$

Thus the dimensionless quantity

$$\chi = m\left(\frac{G}{\hbar c}\right)^{1/2} \qquad (6)$$

measured at different spacetime points can tell us whether masses m are changing. Or alternatively, if we insist on using mass units that are the same everywhere, a change of χ would tell us that the gravitational constant G is changing. We could of course assume that $\hbar$ and c also change. However, by keeping $\hbar$ and c constant we follow the principle of least modification of existing theories. Thus special relativity and quantum theory are unaffected if we keep $\hbar$ and c fixed. This is the conclusion Brans and Dicke arrived at in their approach to Mach's principle. They looked for a framework in which the gravitational constant G arises from the structure of the universe, so that a changing G could be looked upon as the Machian consequence of a changing universe.

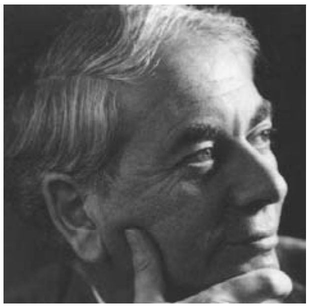
Brans and Dicke looked for a framework in which the gravitational constant G arises from the structure of the universe, so that a changing G could be looked upon as the Machian consequence of a changing universe.

In 1953, D W Sciama (*Figure* 5) had given general arguments leading to a relationship between G and the large-scale structure of the universe. We come across one example of such a relation in the standard Friedmann cosmologies:

$$\rho_0 = \frac{3H_0^2}{4\pi G}q_0. \qquad (7)$$

If we write $R_0 = c/H_0$ as a characteristic length of the universe and $M_0 = 4\pi\rho_0 R_0^3/3$ as the characteristic mass

Figure 5. Dennis W Sciama (1926–2000).



Brans and Dicke postulated that G behaves as the reciprocal of a scalar field ϕ.

of the universe, then the above relation becomes

$$\frac{1}{G} = \frac{M_0}{R_0 c^2} q_0^{-1} \sim \frac{M_0}{R_0 c^2} \sim \sum \frac{m}{rc^2}. \qquad (8)$$

Given a dynamic coupling between the inertia and gravity, a relation of the above type is expected to hold. Note that the last step in (8) assumes that the universal M_0/R_0 is in fact a superposition of individual contributions of the form m/r from all the particles in the universe. Brans and Dicke took this relation as one that determines G^{-1} from a linear superposition of inertial contributions m/rc^2, the typical one being from a mass m at a distance r from the point where G is measured. Since m/r is a solution of a scalar wave equation with a point source of strength m, Brans and Dicke postulated that G behaves as the reciprocal of a scalar field ϕ:

$$G \sim \phi^{-1}, \qquad (9)$$

where ϕ is expected to satisfy a scalar wave equation whose source is all the matter in the universe.

5. The Hoyle–Narlikar Cosmology

We next consider another gravitation theory that may claim to have given the most direct quantitative expression to Mach's principle. This theory was first proposed in 1964 by Fred Hoyle and the author, and we will refer to it here as the HN theory and to the cosmological models based on it as HN cosmologies. Throughout this discussion we will set $c = 1$.

The Hoyle–Narlikar theory arose from the concept of *direct interparticle action*.

Like general relativity and the Brans–Dicke theory, the HN theory is formulated in the Riemannian spacetime. There is one important difference, however, between this theory and all other cosmological theories we have discussed usually. The difference lies in the fact that general relativity, the Brans–Dicke theory, and so on are pure field theories, whereas the HN theory arose from the concept of *direct interparticle action*. The difference between the two types of theories is best seen in the description of electromagnetism. Until the advent of Maxwell's field theory, it was customary to describe electrical and magnetic interactions as instances of direct *action at a distance* between particles. The success of Maxwell's theory established the *field* concept in physics at the expense of the concept of action at a distance (see *Figure* 6).

In the action at a distance scenario the electromagnetic effect from A to B travels into the future and arrives at B *later* than it left A. Such an action is called *retarded* action. The *reaction* from B to A, however travels *back* along the same route, arriving at A *earlier* than it left B. Such an action is called *advanced* action. The presence of both advanced and retarded effects on an equal footing makes the problem non-local; indeed it brings in cosmology.

That cosmological boundary conditions are necessary in the action-at-a-distance framework is seen from the following simple illustration. Any retarded signal emitted by particle a will get an advanced reaction back from b, as shown in *Figure* 7. Thus the theory admits advanced

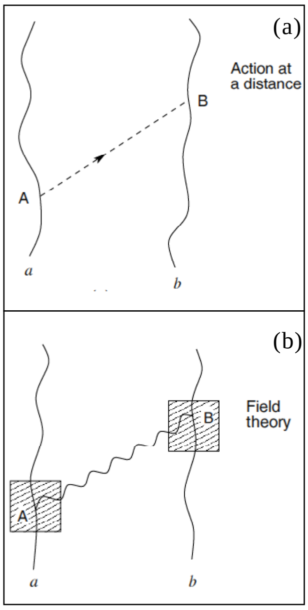


Figure 6. Fields versus action at a distance. In (a) we see points A and B on the worldlines of electric charges 'a' and 'b' interacting directly across a distance provided A and B are connectible by a null ray. In (b) the interaction between A and B proceeds via the electromagnetic field, generated at A through movements of charge 'a' and travelling with the speed of light to B where it conveys the effect of A.

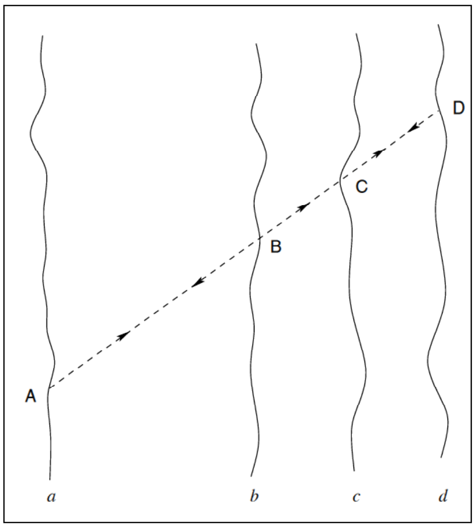


Figure 7. A retarded signal (shown by dotted line) leaving point A on the world line of 'a' hits particles b,c,d,... at points B,C,D,... at later times. Their advanced response returns to A along the same dotted track, no matter how far these particles are from 'a'. Thus even the remote parts of the universe generate instantaneous responses to the retarded disturbance leaving A. In short the response of the whole universe cannot be ignored.

Thus electromagnetism ceases to be a local theory: any so-called local effect must take account of the response of the universe, which consists of reactions from all such particles *b* other than *a*.

signals and appears to violate causality. Moreover, in *Figure* 7 the signal from b arrives at a at the same time that the original signal left a, no matter how far away b is! Thus electromagnetism ceases to be a local theory: any so-called local effect must take account of the response of the universe, which consists of reactions from all such particles b other than a. A 'correct' response can cancel all the acausal effects. This was pointed out first by J A Wheeler and R P Feynman in 1945. Later, between 1962 and 1963, J E Hogarth, F Hoyle, and the author showed that this response depends on the model of the universe. In essence, to produce the correct response the universe must be perfect absorber in the future, i.e., it should be able to absorb all electromagnetic signals directed to the future.

What is the response of the universe? In the 1930s, it had been demonstrated by Dirac that when an electric charge a accelerates, the force of radiative damping that it is subjected to can be calculated by evaluating half the difference of the retarded and the advanced fields of the charge *on its own worldline*:

$$Q(a) = \frac{1}{2}[F^{\mathrm{R}}(a) - F^{\mathrm{A}}(a)]. \tag{10}$$

In the Maxwell field theory, Dirac's result had remained just a curiosity without a proper understanding as to why the radiative reaction must be determined by the above formula. And this was linked with the more basic question that arises when we discuss electromagnetic fields of oscillating system of electric charges. It is customary to choose the retarded solutions of the Maxwell wave equations to describe these fields, on the grounds of causality. And it is because of this choice that the system radiates energy and suffers damping. So the basic question is: Why do we restrict our solutions to the retarded ones and throw away the advanced ones? Or to put is differently, why do we have a principle of causality (that causes *precede* effects), when the basic equations

Why do we restrict our solutions to the retarded ones and throw away the advanced ones?

of physics are time-symmetric?

The Wheeler–Feynman theory provides an answer. The theory is formulated in a time symmetric manner with advanced solutions on equal footing with the retarded ones. Thus a typical particle a generates a 'direct particle field' defined by

$$F(a) = \frac{1}{2}[F^{\mathrm{R}}(a) + F^{\mathrm{A}}(a)], \qquad (11)$$

which is manifestly symmetric with regard to its advanced and retarded components. As seen above, the universe as a whole generates a response to these individual fields of the charges, and in the Wheeler–Feynman theory the 'correct' response from the universe to the motion of a is precisely (10)! It can be shown that for the correct response, the future part of the universe (lying on the future light cone of the radiating system) must be a *perfect absorber* of all retarded, i.e., future-directed signals, and the past part of the universe, an *imperfect absorber* of all advanced, i.e., past-directed signals. In such a universe therefore, if we add this response (10) to the basic time-symmetric field of a, as given by (11), we get the net field in the neighbourhood of a as

$$\begin{aligned} F_{\mathrm{total}}(a) &= F(a) + Q(a) = \frac{1}{2}[F^{\mathrm{R}}(a) + F^{\mathrm{A}}(a)] \\ &+ \frac{1}{2}[F^{\mathrm{R}}(a) - F^{\mathrm{A}}(a)] = F^{\mathrm{R}}(a). \qquad (12) \end{aligned}$$

In this way, we get the total effect in the neighbourhood of a to be a pure retarded one. A correct response therefore eliminates all advanced effects except those present in the radiation reaction. It is interesting (and significant) that the steady state model generates the correct response, while all big bang Friedmann models fail to do so. Because of the crucial requirement of perfect absorption, this theory is sometimes called the 'absorber theory of radiation'.

The Wheeler–Feynman theory provides an answer. The theory is formulated in a time symmetric manner with advanced solutions on equal footing with the retarded ones.

Since Mach's principle (implying as it does a connection between the local and the distant) suggests action at a distance, even an early convert to it like Einstein later became skeptical as to its validity.

Since Mach's principle (implying as it does a connection between the local and the distant) suggests action at a distance, even an early convert to it like Einstein later became skeptical as to its validity. Einstein's objections were based on the belief that action at a distance was supposed to be instantaneous and hence inconsistent with relativity. By the early 1960s, however, it became clear that action at a distance can be made consistent with relativity and also successfully describe electrodynamics besides having interesting cosmological implications. Since Hoyle and the author had played an active role in these developments, they naturally adopted an action-at-a-distance approach to Mach's principle.

Accordingly, we use here the somewhat unfamiliar notation of action at a distance. Let us denote by $a, b, \ldots$ the particles in the universe, with m_a being the mass of the a^{th} particle. As implied by Mach, the mass m_a is not entirely an intrinsic property of particle a; it also owes its origin to the background provided by the rest of the universe. To express this idea quantitatively, write

$$m_a(A) = \lambda_a \sum_{b \neq a} M^{(b)}(A). \tag{13}$$

The above expression means the following. At a typical world point A on the world line of particle a, the mass acquired by a is the net sum of contributions from all other particles $b(\neq a)$ in the universe. The contribution from b at A is given by the scalar function $M^{(b)}(A)$. The coupling constant λ_a is intrinsic to the particle a. Notice, however, that if a were the only particle in the universe, $m_a = 0$ and we have the Machian conclusion arrived at in (4).

A discussion of Mach's principle and alternative cosmologies based on it may be found in the author's textbook *An Introduction to Cosmology* published by Cambridge University Press in 1993.

Address for Correspondence
J V Narlikar
Inter-University Centre for Astronomy & Astrophysics
Ganeshkhind, Post Bag 4
Pune 411 007, India.
Email: jvn@iucaa.ernet.in