THE ASTROPHYSICAL JOURNAL, 455:421–428, 1995 December 20

EINSTEIN'S RADIATION FORMULA AND MODIFICATIONS TO THE EINSTEIN EQUATION

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Received 1993 March 3; accepted 1995 July 3

ABSTRACT

Einstein's radiation formula is supported by the Taylor-Hulse experiment, but its derivation is not self-consistent. Furthermore, as discovered by Einstein, his radiation formula is not compatible with his field equation. As suggested by Einstein's own remark, modifications to the source tensor are necessary. Based on the Taylor-Hulse experiment, in this paper a theory is developed within the theoretical framework of general relativity within which the radiation formula remains the same for binary stars. Concurrently, it is determined that, because of radiation, the source tensor is not zero in a vacuum. Antigravity coupling, suggested by Pauli as a possibility, is a necessary feature. In addition, it is shown that the current theory of linearized gravity is not valid for radiation.

Subject headings: gravitation — radiation mechanisms: nonthermal — relativity

1. INTRODUCTION

General relativity suggests the existence of gravitational waves (Wheeler 1990; Misner, Thorne, & Wheeler 1973). Although such waves have never been directly observed, the Taylor-Hulse experiment supports the existence of energy loss by gravitational radiation (Hulse & Taylor 1975; Wald 1984). While Einstein's radiation formula is supported by the observed data (Hulse & Taylor 1975; Taylor & Weisberg 1982, 1984), one should not consider this to be a verification of Einstein's gravitational radiation theory (see §§ 2 and 3) because his theory does not produce the radiation formula in a self-consistent manner (Wald 1984; Yu 1992). Instead, one should first identify the problems in its derivation and deduce their theoretical implications. Accordingly, to support the radiation formula, one may attempt to develop a self-consistent theory within the theoretical framework of general relativity.

Einstein's radiation formula is based on his notion of energy-momentum components of the gravitational field, a pseudotensor $t_{ab}(g)$ (Pauli 1958). Since his theory is not covariant, doubts have been raised by Lorentz (1916), Levi-Civita (1917), and Einstein (1918) himself. Moreover, it was proven by Denisov & Logunov (1982) and Vlasov & Denisov (1982) that the radiation formula is not an invariant; the rate of energy emission, depending on the choice of the system of coordinates, may be positive, negative, or zero. It seemed that only a covariant theory could be valid in physics (see Appendix). Moreover, linearized gravity necessarily implies the linearized conservation law. This law implies the absence of radiation (Wald 1984; Yu 1992). Thus, it seemed there was little hope to develop a self-consistent theory to support Einstein's formula.

On the question of radiation, theorists seem to be divided into three schools. One school is satisfied with the radiation formula and accepts the limitations of Einstein's theory (Will 1990). Another school would develop a theory within the theoretical framework of general relativity (Wald 1984; Yu 1989). Finally, some have gone so far as to attempt to justify an alternative gravity theory (Logunov & Mestvirishvili 1989). The crucial question—whether Einstein's radiation formula can be supported by a self-consistent theory—has not been answered.

However, as noted by Einstein (Born 1968), linearized gravity is not reliable. Recently, it was found that the principle of equivalence is also a crucial requirement for a physical coordinate system, and therefore a gauge condition may not be valid (Lo 1994). It then became clear that what was a seemingly impossible task would be feasible (see §§ 2, 3, and 4). Although a covariant theory does not produce the same radiation formula, as far as agreement with data, it is sufficient to show that the formula's rates of energy loss, on the time average, are the same.

In this paper, it will be shown that the radiation formula has important implications (see § 3). Because of radiation, the source in a vacuum is necessarily not zero. Antigravity coupling, a possibility pointed out by Pauli (1958), is a necessary feature. The linearized gauge is incompatible with the radiation formula. Moreover, Einstein's radiation formula can indeed be supported with a self-consistent theory within the theoretical framework of general relativity.

2. DERIVATION OF THE RADIATION FORMULA AND SELF-CONSISTENCE

To support Einstein's formula, one must identify the causes of inconsistency in the derivation. Necessary approaches and methods can then be developed for a supporting theory.

Einstein's nonlinear field equation is

$$G_{ab} \equiv R_{ab} - \tfrac{1}{2}Rg_{ab} = -KT_{ab}\,, \tag{1a}$$

where its source KT_{ab} generally depends on the spacetime metric g_{ab}. The harmonic coordinate condition (Misner et al. 1973; Weinberg 1972) is

$$\frac{\partial}{\partial x^a}\left(|g|^{1/2}g^{ab}\right) = 0\,, \tag{1b}$$

where g is the determinant of the metric. For weak gravity, it is convenient to consider equations expressed in terms of deviations γ_{ab} ($\equiv g_{ab} - \eta_{ab}$) from the flat metric, in which case equations (1a) and (1b) are respectively linearized to (Misner et al. 1973; Wald 1984)

$$G_{ab}{}^{(1)} = -KT_{ab}\,, \tag{2a}$$

where

$$G_{ab}{}^{(1)} \equiv \tfrac{1}{2}\partial^c\partial_c\bar{\gamma}_{ab} + H_{ab}{}^{(1)}\,,$$
$$H_{ab}{}^{(1)} \equiv -\partial^c\partial_{(b}\bar{\gamma}_{a)c} + \tfrac{1}{2}\eta_{ab}\,\partial^c\partial^d\bar{\gamma}_{cd}\,;$$
$$\partial^a\bar{\gamma}_{ab} = 0\,, \tag{2b}$$

where

$$\bar{\gamma}_{ab} \equiv \gamma_{ab} - \tfrac{1}{2}\eta_{ab}\gamma\ , \quad \gamma \equiv \eta^{ab}\gamma_{ab}\ .$$

The linearized "gauge" (eq. [2b]) reduces equation (2a) to

$$\tfrac{1}{2}\partial^c\partial_c\,\bar{\gamma}_{ab} = -KT_{ab}\ . \tag{2c}$$

It thus follows from equation (2b) that the linearized conservation law,

$$\partial^a T_{ab} = 0\ , \tag{2d}$$

is necessarily *exactly* satisfied. Note that equation (2d) is also implied directly by equation (2a) since $\partial^a G_{ab}{}^{(1)} \equiv 0$.

The effective stress-energy tensor of the gravitational field, valid to second order, is assumed to be (Wald 1984; see also Appendix)

$$t_{ab} = G_{ab}{}^{(2)}/K\ , \tag{3a}$$

where

$$G_{ab}{}^{(2)} = G_{ab} - G_{ab}{}^{(1)}\ . \tag{3b}$$

Then the rate of energy loss due to radiation is (Wald 1984)

$$-\frac{dE}{dt} = \int t_{k0}\,dS^k = \frac{1}{K}\int G_{k0}{}^{(2)}\,dS^k\ , \tag{3c}$$

where

$$E = \int t_{00}\,d^3x + \int T_{00}\,d^3x\ ,$$

in which expression the first integral term is the total energy associated with γ_{ab}. Note that E is unchanged if the Landau-Lifshitz "pseudotensor" is used in equation (3c). Also, equation (3c) requires only that

$$\int [\partial^a G_{a0}{}^{(2)} + \partial^a T_{a0}]\,d^3x = 0\ .$$

To evaluate formula (3c), one must solve equation (2c) to obtain

$$\bar{\gamma}_{ab}(x^i, t) = -\frac{K}{2\pi}\int \frac{1}{R}\,T_{ab}[y^i, (t-R)]\,d^3y\ , \tag{4a}$$

where

$$R^2 = \sum_{i=1}^{3}(x^i - y^i)^2\ .$$

In the field far from the source, equation (4a) can be approximated by

$$\bar{\gamma}_{ab}(x^i, t) = -\frac{K}{2\pi}\frac{1}{r}\int T_{ab}[y^i, (t-R)]\,d^3y\ , \tag{4b}$$

where

$$r^2 = \sum_{i=1}^{3}(x^i)^2\ .$$

Then one can use equation (2d) to establish (Wald 1984)

$$\int T^{jk}\,d^3x = \frac{1}{2}\frac{\partial^2}{\partial t^2}\int T^{00}x^k x^j\,d^3x\ . \tag{5}$$

Substituting equation (5) into equation (4b), one obtains

$$\bar{\gamma}^{jk} = -\frac{K}{4\pi}\frac{1}{r}\frac{\partial^2}{\partial t^2}\int T^{00}x^k x^j\,d^3x\ . \tag{6}$$

Equation (6) manifests that the metric is periodic for periodic motions.

Then the rate-of-energy-loss formula (eq. [3c]) becomes

$$-\frac{dE}{dt} = \frac{G}{45}\,(\dddot{q}_{kj}\,\dddot{q}^{kj}) \geq 0\ , \tag{7}$$

where q_{jk} is the quadrupole moment tensor of the material system. Equation (7) is the famed "quadrupole radiation" formula. In this derivation equations (2c), (2d), and (3c) are the independent equations.

However, Einstein's theory is not self-consistent. As pointed out by Wald (1994) and Yu (1992), the linearized conservation law (eq. [2d]) implies that "two stars would not orbit each other but would move on geodesics of the flat metric." This means $\ddot{q}_{jk}$ is zero, and therefore there is no gravitational radiation. The usual formula for the rate of change of orbital period has been derived assuming equation (7) without reference to equation (2d), and the analysis by Peters & Mathews (1963) is based on Newtonian orbits. That derivation is illegitimate, as equation (7) has been derived from equation (2d). The claim that observed data from PSR 1913+16 has verified Einstein's gravitational radiation theory is therefore groundless (Yu 1992).

Nevertheless, this conclusion was not generally accepted (Will 1990), and some conjectured that this could be a matter of an appropriate approximation for equation (1a). The futility of such efforts is manifested in a concluding remark of Damour's (1987) that "nearly all aspects of approximation methods need to be thoroughly re-investigated." As noted by Einstein (Born 1968) in 1936, equations (1a) and (2c) are not compatible.

To have radiation, as pointed out by Wald (1994), one must obtain a gravitational acceleration. From the conservation law,

$$0 = \nabla_a T^{ab} = \partial_a T^{ab} + \Gamma^b_{ac}\,T^{ac} + \Gamma^a_{ac}\,T^{cb}\ , \tag{8}$$

one can see that, for a first-order approximation of the metric, the conservation law is accurate up to the second order. Thus, it is possible to describe gravitational radiation based on a first-order approximation of the metric. On the other hand, one must also show that the linearized conservation law (eq. [2d]) is not necessary. In the derivation, equation (2d) is used only to obtain equation (5). Because of weak gravity, equation (5) can be derived with $\nabla^a T_{ab} = 0$. In this alternative derivation, the accuracy of $G_{ab}{}^{(2)}$, up to second-order deviations, remains the same. Thus, equation (2d) is indeed not needed to obtain the radiation formula.

However, equation (2a) directly implies equation (2d). Moreover, although Einstein's formula is based on the subsequent equation (2c), the "gauge condition" (eq. [2b]) still implies equation (2d). Thus, his radiation formula is not only independent of, but inconsistent with, linearized gravity. Therefore, a necessary implication of his formula is that equation (2c) should be justifiable without equation (2b). Then the analysis by Peters & Mathews (1963) becomes valid.

Because observation supports equation (7) and, therefore, equation (2c), it is necessary to modify equation (1a). Then one must show also that such a modification is compatible with equation (3c). This can be done within the theoretical framework of general relativity.

3. MODIFICATIONS TO THE EINSTEIN EQUATION

The existence of gravitational waves is due to physical considerations that are independent of Einstein's equation (Wheeler 1990). Since such waves should carry energy-momentum (Stephani 1990), an expected modification would be that the source term should not be zero in a vacuum. This means that a source tensor due to the gravity energy-stress tensor must exist. Moreover, such a tensor should have an antigravity coupling since gravity would not be self-generating. The radiation formula precisely confirms these conclusions. In other words, while his radiation formula supports the Einstein tensor, it is necessary to modify the source tensor.

Although Einstein was aware of the inconsistency between his equation and his formula (Born 1968), because of a lack of experimental evidence, he could not resolve this difficulty. However, as if to encourage a modification, Einstein once remarked that the left-hand side of his equation was granite, but the right-hand side (the source) was sand (Lo 1992; Kalligas, Wesson, & Everitt 1992).

In view of the fact that there is no existing gravity energy-stress tensor, it seems simple and natural to assume that the source tensor T_{ab} is zero in a vacuum. Now let us show that such a current version of Einstein's field equation is not consistent with Einstein's radiation formula.

The Einstein equation can be written in an alternative form,

$$G_{ab}^{(1)} = -K\tilde{T}_{ab} = -K(T_{ab} + t_{ab}) , \tag{9a}$$

where $t_{ab} = G_{ab}^{(2)}/K$. Then

$$\partial^a \tilde{T}_{ab} = 0 \tag{9b}$$

is exact since $\partial^a G_{ab}^{(1)} \equiv 0$. If the linearized gauge, $\partial^a \bar{\gamma}_{ab} = 0$ (eq. [2b]), is applied, then equation (9a) is reduced to

$$\tfrac{1}{2}\partial^c \partial_c \bar{\gamma}_{ab} = -K\tilde{T}_{ab} . \tag{9c}$$

A formal "solution" to equation (9c) would be

$$\bar{\gamma}_{ab}(x^i, t) = -\frac{K}{2\pi}\int \frac{1}{R}\,\tilde{T}_{ab}[y^i, (t - R)]\,d^3y , \tag{9d}$$

where

$$R^2 = \sum_{i=1}^{3} (x^i - y^i)^2 .$$

In the right-hand side of equation (9d), the metric is implicitly included. Now, it seems, equation (9b) implies that there is no difficulty in accommodating the linearized gauge (eq. [2b]).

However, the problem of compatibility with Einstein's radiation formula remains unless the contribution of T_{ab} dominates the effect of t_{ab} $(= G_{ab}^{(2)}/K)$. If KT_{ab} is nonzero only in a finite region, then it is not clear whether the contribution of Kt_{ab} $(= G_{ab}^{(2)}$, which may be nonzero almost everywhere) is negligible. Now one can see that the contribution of a radiating T_{ab} may *not* be dominating.

It will be shown that, according to equation (4a), the contribution of $G_{ab}^{(2)}$ diverges. At large r, we have approximately (Misner et al. 1973; Wald 1984)

$$G_{ab}^{(2)} + H_{ab}^{(1)} \sim O(r^{-2}) . \tag{9e}$$

The contribution of $[G_{ab}^{(2)} + H_{ab}^{(1)}]$ to $\bar{\gamma}_{tt}(x^i, t)$ would be

$$D_{tt}(x^i, t) = -\frac{1}{2\pi}\int \frac{1}{R}\,[G_{tt}^{(2)} + H_{tt}^{(1)}]\,d^3y = -\frac{1}{2\pi}\left(\int_{r\le a} + \int_{r>a}\right) . \tag{9f}$$

Note that

$$\int_{r>a} \sim \int_{r>a} \frac{d\Omega}{r}\left(\frac{1}{r^2}\right) r^2\,dr = 4\pi \int_{r>a} \frac{dr}{r} \tag{9g}$$

for large a and x^i near the origin. Thus, equation (9f) may not be mathematically negligible if the source has been emitting waves long enough (Misner et al. 1973), and divergence would occur so long as the source were nonzero only in a finite region. Moreover, since such a divergence has nothing to do with the emission process (Misner et al. 1973), the approximation is not valid in physics.

Thus, the problem of compatibility can be resolved only in terms of physics. The divergent contribution must be canceled by an additional source tensor, which must be of second order and nonzero almost everywhere in a vacuum. From the viewpoint of physics, this should be the energy-stress tensor $t(g)_{ab}$ for gravity. These conditions suggest that the modified Einstein equation would be $G_{ab} = -K[T_{ab} - t(g)_{ab}]$. Tensor $t(g)_{ab}$ should have the antigravity coupling because gravity should not be self-generating. Thus, the radiation formula implies that the assumption of a zero source in vacuum is incorrect, and the current Einstein equation is problematic for questions related to radiation.

Moreover, if the metric is periodic, then a certain time average of $G_{a0}^{(1)}$ is zero. It follows from equation (1a) that the time average of $-G_{a0}^{(2)}$ is the time average of KT_{a0}. Equation (3c) implies that the time average of $G_{\check{r}0}^{(2)}$ (where $\check{r}$ is the radial direction) is nonzero in vacuum. This is supported by Einstein's own conclusion that, based on exact solutions of his field equation, there is no radiation (Born 1968). Thus, equation (3c) implies that the source T_{ab} cannot be zero in vacuum. This is also supported by the fact that, in the literature (Kramer et al. 1980), there is no exact physical solution of radiation when the source tensor in vacuum is assumed to be zero.

In conclusion, Einstein's radiation formula implies that his field equation must be modified so that the source tensor is nonzero in vacuum. The detailed theoretical implications are the following:

1. The effective stress-energy tensor of the gravitational field, $t_{ab}(g)$, is actually a tensor. The assumption that $G_{ab}^{(1)} = 0$ in vacuum is equivalent to

$$G_{ab} = G_{ab}^{(1)} + G_{ab}^{(2)} = Kt_{ab}(g) . \tag{10}$$

Therefore, $t_{ab}(g)$ is actually a tensor although its approximation appears in equation (3a) as a pseudotensor. This means that the *covariant* nature of general relativity is maintained.

2. The coupling of $t_{ab}(g)$ is antigravitational. Equation (10) means that the *factual* assumption in vacuum is

$$T_{ab} = -t_{ab}(g) . \tag{11}$$

Equation (11) means that the tensor t_{ab} has *antigravity* coupling. Pauli (1958) pointed out that general relativity does not provide a physical interpretation for the sign of the gravitational coupling constant. Thus, in principle, antigravity coupling is allowed. This is necessary due to the radiation formula. Moreover, antigravity coupling is supported by the fact that the Einstein tensor of a gravitational plane wave has, on the average, a different sign from that of the massive matter (Lo 1992, 1994).

3. Equations (10) and (11) imply that, in general, the Einstein equation must be extended to the following form:

$$R_{ab} - \tfrac{1}{2}Rg_{ab} = -K[T_{ab}(m) - t_{ab}(g)] = -KT_{ab} , \quad (12a)$$

where $T_{ab}(m)$ is the stress tensor for massive matter and $t_{ab}(g)$ is for the field energy. The term t_{ab}, as indicated in equation (9d), cancels the divergent integral. Now it is clear that, if the stress tensor T_{ab} is zero in vacuum, then it is not possible to have gravitational radiation since $t_{ab} = 0$ (see also Yu 1989; Yilmaz 1958; Born 1968). Note that equation (12a) extends the suggestion of Lorentz (1916) and Levi-Civita (1917).

4. Due to energy-momentum conservation, equation (12a) implies that

$$\nabla^a T_{ab}(m) = 0 , \quad (12b)$$

$$\nabla^a t_{ab}(g) = 0 \quad (12c)$$

because of their coupling sign difference. Thus, for a pointlike particle, the equation of motion is still a geodesic equation.

Note that $t_{ab}(g)$, being an energy-stress tensor, is not a geometrical part. In the literature, based on different theoretical considerations, there are theories (Yilmaz 1992; Yu 1989; Brans & Dicke 1961) in which a nonmatter term is present in the source but without antigravity coupling. Note that both the presence of the gravitational energy-stress $t_{ab}(g)$ and its antigravity coupling are necessary as a result of the Taylor-Hulse experiment.

Since the linearized gauge (eq. [2b]) is not valid, according to equation (2c), one obtains

$$Kt_{ab}(g) \approx G^{(2)}{}_{ab} + H^{(1)}{}_{ab} . \quad (13)$$

Equation (13) implies that equation (3c) would be modified. However, if the motion is periodic, on the time average, the tensor component t_{0k} $[k = (x, y, z)]$ remains essentially $G^{(2)}{}_{0k}/K$ as assumed earlier.

4. MAXWELL-NEWTON APPROXIMATION AND EINSTEIN'S RADIATION FORMULA

It remains to justify equation (2c). Physically, equation (2c) gives the direct influence of the massive source to the field. Whereas the right-hand side of equation (13) represents the gravitational self-interaction, $t_{ab}(g)$ is the gravitational energy-stress. Equation (2c) implies also that a gravitational wave propagates with the speed of light. Given a particle moving along a geodesic, equation (2c) is the natural extension from Newtonian theory. For clarity, equation (2c) is written as

$$\partial^c \partial_c \bar{\gamma}_{ab}{}^{(1)} = -2KT_{ab}(m) , \quad (14)$$

where $\bar{\gamma}_{ab}{}^{(1)}$ $[= \bar{\gamma}_{ab} - \bar{\gamma}_{ab}{}^{(2)}]$ is an approximation of the first-order deviations, $\bar{\gamma}_{ab}{}^{(2)}$ is of second order, and $T_{ab}(m)$ is the energy-stress tensor for massive matter. Note that equation (14) is now an approximation of equation (12). [In linearized gravity, T_{ab} would be used instead of $T_{ab}(m)$.]

Obviously, equation (14) is an approximation after the coordinate system has been chosen. The asymptotic flatness of the metric is the implicit gauge. Mathematically, as shown above, equation (14) manifests the necessary approximate cancellation of the second-order terms and, therefore, is not a simple linearization. For the case of an electromagnetic plane wave (Lo 1994), equation (14) is exact since $T_{ab}(m) = 0$.

Moreover, equation (14) is justified on its agreement with experiments (Will 1990; Anderson et al. 1990; Prince et al. 1991; Wolszan 1991; Ohanian & Ruffini 1994). For a static mass distribution, it produces Newton's law of gravity. For nonstatic cases, it produces Einstein's radiation formula. To be distinct from linearized gravity, equation (14) will be called the Maxwell-Newton approximation. The validity of this approximation will be further tested in the Stanford Gyroscope experiment (Kalligas, Wesson, & Everitt 1995, 1992; Muhlfelder et al. 1995).

Having made clear the underlying physics, it remains to show that such an approach provides the required approximation, as follows:

1. For self-consistency, it is necessary that, according to equation (12a), equation (14) indeed gives a first-order approximation.

2. To support a radiation formula, equation (14) must imply, to second order, $\partial_a t^{ad} \approx 0$. This is also required by equation (12c).

Note that condition (1) implies also that $\nabla_a T^{ad}(m) = 0$ is satisfied to second order.

It follows from equation (14) that

$$\tfrac{1}{2}\partial^c \partial_c L_b = -K\partial^a T_{ab}(m) , \quad L_b = \partial^a \bar{\gamma}_{ab} . \quad (15a)$$

Since $\nabla^a T(m)_{ab} = 0$, $K\partial^a T(m)_{ab}$ are second-order deviations. It follows from equation (15a) or (4a) that L_b are also second-order. This implies, from equation (12a), that, up to first-order deviations,

$$\tfrac{1}{2}\partial^c \partial_c \bar{\gamma}_{ab} = -K[T_{ab}(m) - t_{ab}] . \quad (15b)$$

Physically, since t_{ab} is induced by $T_{ab}(m)$, Kt_{ab} would be of second order. If equation (14) is a valid approximation, Kt_{ab} has to be of second order. Thus, equation (14), as a first-order approximation, is consistent with equation (12a).

It will be shown that $K\partial^a t_{ab}$ is of third order. Let us denote

$$R_{ab} = R_{ab}{}^{(1)} + R_{ab}{}^{(2)} , \quad R^{(i)} = \eta^{cd} R_{cd}{}^{(i)} , \quad (16)$$

where

$$R_{ab}{}^{(1)} = \tfrac{1}{2}\partial^c \partial_c \gamma_{ab} - \tfrac{1}{2}(\partial^c \partial_b \bar{\gamma}_{ac} + \partial^c \partial_a \bar{\gamma}_{bc}) ,$$

$$R_{ab}{}^{(2)} \approx \tfrac{1}{2}\partial^d \gamma^c{}_a(\partial_c \gamma_{db} - \partial_d \gamma_{bc}) - \tfrac{1}{2}\gamma^{cd}\partial_a \partial_b \gamma_{cd} - \tfrac{1}{4}\partial_b \gamma^{cd}\partial_a \gamma_{cd} + \tfrac{1}{2}\gamma^{dc}\partial_c(\partial_b \gamma_{da} + \partial_a \gamma_{db} - \partial_d \gamma_{ab})$$

up to second-order deviations. It follows from equation (16) that

$$R = R^{(1)} + R^{(2)} - \gamma^{cd}[R_{cd}{}^{(1)} + R_{cd}{}^{(2)}] , \quad (17a)$$

$$G_{ab}{}^{(1)} = R_{ab}{}^{(1)} - \tfrac{1}{2}\eta_{ab} R^{(1)} , \quad (17b)$$

$$G_{ab}{}^{(2)} = R_{ab}{}^{(2)} - \tfrac{1}{2}\eta_{ab} R^{(2)} + \tfrac{1}{2}\eta_{ab}\gamma^{cd} R_{ab}{}^{(1)} - \tfrac{1}{2}\gamma_{ab} R^{(1)} + \tfrac{1}{2}[\![\eta_{ab}\gamma^{cd} R_{cd}{}^{(2)} - \gamma_{ab}\{R^{(2)} - \gamma^{cd}[R_{cd}{}^{(1)} + R_{cd}{}^{(2)}]\}]\!] . \quad (17c)$$

Lengthy but straightforward calculation shows that, up to second order,

$$\begin{aligned} \partial_c G^{cd} &\approx [\partial^a G_{ab} - \gamma^{ca}\partial_c G_{ab}{}^{(1)} - \tfrac{1}{2}\partial^a \gamma G_{ab}{}^{(1)}]\eta^{bd} - G_{ab}{}^{(1)}\partial^a \gamma^{bd} \\ &\approx (\tfrac{1}{2}\partial_b \gamma^{ca} \tfrac{1}{2}\partial^k \partial_k \bar{\gamma}_{ac} - \tfrac{1}{2}\partial^a \gamma \tfrac{1}{2}\partial^k \partial_k \bar{\gamma}_{ab})\eta^{bd} - \tfrac{1}{2}\partial^k \partial_k \bar{\gamma}_{ab}\partial^a \gamma^{bd} \\ &\approx (\tfrac{1}{2}\partial_c \gamma^{ab}\eta^{cd} - \tfrac{1}{2}\partial^a \gamma \eta^{bd} - \partial^a \gamma^{bd})[-KT(m)_{ab}] . \end{aligned} \quad (18)$$

It follows from equation (12a) that equation (18) implies, to second order,

$$\partial_a T(m)^{ad} + \Gamma^d_{ab} T(m)^{ab} + \Gamma^a_{ab} T(m)^{db} \approx \partial_a t^{ad} , \quad (19a)$$

$$\partial_a t^{ad} \approx 0 . \quad (19b)$$

Thus, it has been proven that the Maxwell-Newton approximation is a self-consistent approach with valid physical justifications. In particular, it unequivocally supports Einstein's radiation formula as a consequence of general relativity.

Since $\partial^a G_{ab}{}^{(1)} \equiv 0$, equation (12) implies

$$G_{ab}{}^{(2)}/K + T_{ab}(m) = t_{ab}(g) - G_{ab}{}^{(1)}/K , \quad (20a)$$

$$\partial^a G_{ab}{}^{(2)}/K + \partial^a T_{ab}(m) = \partial^a t_{ab}(g) . \quad (20b)$$

For weak gravity, because $\partial^a t_{ab} \approx 0$ up to second order, $\partial^a G_{ab}{}^{(2)}$ would relate mainly to the energy-momentum of matter as its source while, in vacuum, $\partial^a G_{ab}{}^{(2)}/K$ is equal to $\partial^a t_{ab}(g)$. In other words, gravity energy and the motion of particles influence each other mainly through geometry.

It follows from equation (20) that, approximately,

$$-\frac{dE}{dt} = \int t_{a0}\, dS^a = \frac{1}{K}\int G_{a0}{}^{(2)}\, dS^a - \frac{1}{2K}\int (\partial_0 L_a + \partial_a L_0)\, dS^a . \quad (21)$$

Note that the final integral term comes from $\partial^a G_{ab}{}^{(1)} \equiv 0$. Since such a relation is independent of the physical process, from the viewpoint of physics, the second integral is irrelevant. In fact, based on equation (6), calculation shows that the time average of the second integral is zero. Then, equation (21) is reduced back to equation (3c).

In summary, given the assumption that Einstein's radiation formula is valid, modifications on the source tensor are necessary, as predicted by Einstein's remark. Remarkably, his radiation formula means that in vacuum the source tensor is not zero and that its coupling is antigravitational. After the necessary modifications, the radiation formula remains the same. Thus, this modification process is self-consistent.

5. CONCLUSION AND DISCUSSION

It is interesting that a self-consistent covariant theory can be developed to support Einstein's radiation formula. Some colleagues may have linked the validity of "gauge" and linearized gravity with his radiation formula. However, the fact is that both linearized gravity and the "gauge" are inconsistent with his formula. The most important conclusion is, though, that Einstein's radiation formula *requires* modification of his field equation. A crucial starting point of this analysis is the observation that the gauge may not be applicable for gravitational waves. (In electrodynamics, classical gauge is also not valid; Tonomura et al. 1986; Aharonov & Bohm 1959.) The inadequacy of this notion is recently proven (Lo 1994) because it can be inconsistent with relativistic causality and the principle of equivalence.

The source tensor in vacuum was controversial (Yu 1989; Yilmaz 1958). Now, based on Einstein's radiation formula, T_{ab} is determined to be nonzero in a vacuum. Consequently, T_{ab} must consist of two parts: a massive tensor $T_{ab}(m)$ and a gravity energy-stress tensor $t_{ab}(g)$. Now it is clear that $t_{ab}(g)$ only appears to be a pseudotensor because of approximation. Physically, since $t_{ab}(g)$ is induced by $T_{ab}(m)$, $t_{ab}(g)$ should, in agreement with this analysis, have a coupling sign that is opposite to the coupling sign of $T_{ab}(m)$. Such an antigravity coupling is further confirmed by gravitational plane waves (Lo 1992, 1994).

The main equation (eq. [12]) is within the theoretical framework of general relativity since general relativity does not specify the coupling constant or the form of an energy-stress tensor (Pauli 1958). (One may recall that the initial field equation was $R_{ab} = -KT_{ab}$ in general relativity.) Moreover, as a version of Einstein's field equation, equation (12) is covariant and can be derived from a Lagrangian function with the variational principle.

It should be pointed out that $t_{ab}(g)$, being an energy-stress tensor, is not a geometrical part although $t_{ab}(g)$ involves only gravity. Moreover, tensor $t_{ab}(g)$ is not a physical cause of the metric, while $Kt_{ab}(g)$ is a source term in Einstein's equation. A physical cause and a mathematical source term can be different (Lo 1994), although they are the same in electrodynamics. In general relativity, an energy-stress tensor involves the metric and, therefore, cannot be a cause of the metric. Thus, $t_{ab}(g)$ is a valid source term in Einstein's field equation. Note that equation (12a) extends the proposal made by Lorentz (1916) and Levi-Civita (1917).

In short, this modification is necessary if Einstein's radiation formula is valid. Also, this modification is self-consistent.

The Maxwell-Newton approximation (eq. [14]) is not generally covariant and is an approximation after the coordinate system has been chosen. Equation (14) is based on physics instead of the notion of gauge. From the viewpoint of physics, this is appropriate. As pointed out by Wheeler (1990), radiation is due to the fact that gravity travels at a finite speed and should not be inextricably related to a notion in pure mathematics. It seems, once equation (14) is accepted, in this new derivation, problems related to the linearized conservation law no longer exist. Damour (1987) has concluded that nearly all aspects of approximation methods need to be thoroughly reinvestigated. Without $t_{ab}(g)$, as shown in § 3, Einstein's field equation is inconsistent with Einstein's radiation formula. Thus, this analysis also supports Damour's conclusion.

For the static case, the Maxwell-Newton approximation coincides with linearized gravity. Thus, the Maxwell-Newton approximation produces Newton's law and is supported by experiments (Anderson et al. 1990; Prince et al. 1991; Wolsean 1991; Ohanian & Ruffini 1994). The verification of Einstein's radiation formula can be considered the strongest evidence supporting the Maxwell-Newton approximation. Moreover, there could be more evidence to establish the Maxwell-Newton approximation directly—because of the superficial mathematical similarity between the Maxwell-Newton approximation and the "linearized" Einstein equation (due to the cancellation of two errors), existing theories to test linearized gravity can be used to check the Maxwell-Newton approximation. For instance, magnetic gravitational effects would provide a good test. An experiment in the near future is the Gravity Probe B gyroscopes (Muhlfelder et al. 1995), which measure the change of the spinning direction.

In comparison with the massive tensor $T_{ab}(m)$, the field tensor $t_{ab}(g)$ is of higher order. For static cases, this analysis implies that $t_{ab}(g)$ zero in vacuum would be a good approximation. This would mean that the effects of this modification will not normally be observed under most circumstances. Therefore, this modification would not noticably change the existing agreement between theory and previous observations.

However, further investigations on strong gravity can be done only after the exact form of $t_{ab}(g)$ is determined. Since Einstein's radiation formula is well supported by observations, it is expected that equation (12) would give a more accurate physical description for verified predictions and may provide a more complete picture for unverified predictions because there is a gravity energy-stress tensor $t_{ab}(g)$.

Now, it seems, many of the severe criticisms (Logunov & Mestvirishvili 1989) of general relativity have become meaningless. These are as follows:

1. Einstein's quadrupole formula for gravitational radiation is not a corollary of general relativity.
2. When a gravitational field and matter are taken in conjunction, the general theory of relativity has not, and cannot have, energy-momentum conservation laws.
3. It does not follow from general relativity, in principle, that a double star loses its energy by gravitational radiation.
4. General relativity does not have the classical Newtonian limit, and hence, it does not satisfy a fundamental physical principle, the correspondence principle.

However, this theory is not exactly complete since equation (14) is only an approximation. It remains to find an exact expression for the gravity tensor $t_{ab}(g)$. Nevertheless, this theory does justify Einstein's radiation formula. In view of the absence of a commonly accepted theory, the present analysis may serve as an interim theory to be completed.

The focus of this paper is limited to the necessary implications of the Hulse-Taylor experiment. To address the possible exact forms of the gravitational energy-stress tensor $t_{ab}(g)$ requires extensive and thorough considerations. This is beyond the scope of this paper. This analysis has established a new criterion for the validity of a gravity theory if it includes the Einstein tensor in its field equation. This means that the existing theories of Brans & Dicke (1961), Moffat (1984), Yilmaz (1992), Yu (1958), and others can be further examined with this criterion. I believe that at present such considerations should be left to those authors themselves. Such discussions would be more appropriate when the possible exact forms of $t_{ab}(g)$ are considered.

The author gratefully acknowledges stimulating discussions with Professors P. Morrison, R. M. Wald, and Xin Yu and valuable comments from Professors J. A. Wheeler and H. Yilmaz. This work is supported in part by The Chen Foundation and Innotec Design, Inc., U.S.A.

APPENDIX

THE GRAVITATIONAL ENERGY-STRESS TENSOR

In a field theory, a central problem is the exchange of energy between a particle and the field where the particle is located. Because a particle gains energy from or loses energy to the field, the field energy-stress has to be *localized*. This is independent of the specific form of an energy conservation law.

The energy-momentum conservation law is usually related to the equation of motion for a particle. In the spirit of Faraday and Maxwell, the conservation law would be written in the following form:

$$\partial_\mu[T(p)^{\mu\nu} + T(g)^{\mu\nu} + T(E)^{\mu\nu} + \cdots] = 0 , \tag{A1}$$

where $T(p)^{\mu\nu}$, $T(g)^{\mu\nu}$, and $T(E)^{\mu\nu}$ are respectively the energy-stress tensors of the particle, gravity, and electromagnetism. In general relativity, the partial derivative ∂_μ is supposedly just replaced by the covariance derivative ∇_μ. This is, indeed, the case for electromagnetism. However, the terms $\partial_\mu[T(p)^{\mu\nu} + T(g)^{\mu\nu}]$ are actually replaced by only $\nabla_\mu T(p)^{\mu\nu}$. Thus, since geometry is involved in gravity, the question of a gravitational energy-stress tensor is not a straightforward matter.

The equivalence principle implies that gravity is a manifestation of the metric and that a neutral particle follows a geodesic. The geodesic equation is

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha{}_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0 , \tag{A2}$$

where

$$\Gamma^\alpha{}_{\mu\nu} = \frac{g^{\alpha\beta}}{2}\left(\frac{\partial g_{\beta\mu}}{\partial x^\nu} + \frac{\partial g_{\beta\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta}\right)$$

are the Christoffel symbols and λ is a parameter. The fact that $\Gamma^\alpha{}_{\mu\nu}$ is a pseudotensor leads to the viewpoint that the gravitational energy-stress is not localizable (Misner et al. 1973) or even that there is no energy conservation law in general relativity (Logunov & Mestvirishvili 1989). While the latter view explicitly proclaims the invalidity of general relativity, the former view attempts to defend relativity. However, since physics requires a field energy-stress to be localized, the effect of this defense is actually destructive.

The nonlocalizable argument is as follows: The equivalence principle implies that all the Christoffel symbols are zero in a local Minkowski space. This implies the absence of gravitational force, and no force means no energy. First of all, in general relativity, there is no gravitational force. When all the Christoffel symbols are zero, the observed effect is no acceleration. However, no acceleration does not imply a nonlocalized energy-stress. For example, consider an electron in an electromagnetic field. The field energy-stress is localized, but in the reference frame that the electron carries, the electron has no acceleration. Thus, the nonlocalizable argument based on the equivalence principle is not valid.

Moreover, in *Relativity and Problems of Space* (1954), Einstein added the crucial phrase, "at least to a first approximation" on the indistinguishability between gravity and acceleration (Einstein 1982). Note that whereas a geodesic equation requires only first-order derivatives of the metric, the Einstein tensor requires second-order derivatives. Eddington (1923) also pointed out the limitation of the equivalence principle. He wrote that

> there are more complex phenomena governed by equations in which the curvatures of the world are involved; terms containing these curvatures will vanish in the equations summarizing experiments made in a flat region, and would have to be reinstated in passing to the general equations. Clearly there must be some phenomena of this kind which discriminate between a flat world and a curved world; otherwise we could have no knowledge of world-curvature. For these the Principle of Equivalence breaks down.

In short, both Einstein and Eddington recognized that the equivalence principle can be compatible with a localized energy-stress tensor.

Since Einstein's notion of gravitational energy-stress is a pseudotensor, it can only be an approximation for some coordinate systems. Einstein's radiation formula is expressed in terms of the quadrupole moments of a Cartesian coordinate system. Moreover, the agreement with data is only on the time average. Thus, experiment supports only the time average of an integral that is based on Einstein's notion. In other words, the gravitational energy-stress can be different from his.

Einstein believed that the gravitational field together with matter must obey a conservation law of some kind. However, his derivation is based on intuition rather than a physical principle (Einstein did not realize that, because geometry is involved in gravity, such a conservation law may not be as straightforward as equation [A1].) Moreover, from his derivation, his notion of gravitational energy-stress is by no means unique. The Einstein tensor can be written as

$$gG^{ab} = \partial_c h^{abc} + Kg\tau^{ab} , \tag{A3}$$

where $h^{abc} = -h^{acb}$ and g is the determinant of the metric. Then $G_{ab} = -KT_{ab}$ isequivalent to

$$-g(T^{ab} + \tau^{ab}) = \partial_c h^{abc} , \tag{A4}$$

which implies

$$\partial_a[g(T^{ab} + \tau^{ab})] = 0 . \tag{A5}$$

Equation (A4) can be written alternatively as

$$\sqrt{-g}(T^a{}_b + \tau^a{}_b) = \partial_c \sigma^{ca}{}_b . \tag{A6}$$

Obviously, τ^{ab} depends on the choice of h^{abc} (or $\sigma^{ca}{}_b$). If

$$\sigma^{na}{}_b = \frac{g_{bm}}{\sqrt{-g}} \partial_l[-g(g^{ma}g^{nl} - g^{mn}g^{al})]/2 , \tag{A7a}$$

then we arrive at the Einstein pseudotensor,

$$K\tau^a{}_b = \sqrt{-g}\,[-2\Gamma^a{}_{ml}\Gamma^l{}_{bp}g^{mp} + \Gamma^l{}_{ml}\Gamma^m{}_{bp}g^{ap} + \Gamma^l{}_{lm}\Gamma^a{}_{bp}g^{mp} + \Gamma^l{}_{bl}\Gamma^a{}_{mp}g^{mp} - \Gamma^l{}_{bl}\Gamma^p{}_{mp}g^{ma} - \delta^a{}_b(g^{mp}\Gamma^l{}_{mp}\Gamma^n{}_{ln} - g^{nl}\Gamma^p{}_{ml}\Gamma^m{}_{pn})]/2 . \tag{A7b}$$

If

$$h^{abc} = \partial_m[-g(g^{ab}g^{mc} - g^{ac}g^{mb})]/2 , \tag{A8a}$$

then we arrive at the Landau-Lifshitz symmetric pseudotensor,

$$K\tau^{ab} = [(2\Gamma^n{}_{ml}\Gamma^p{}_{np} - \Gamma^n{}_{ml}\Gamma^p{}_{np} - \Gamma^n{}_{ml}\Gamma^p{}_{np})(g^{al}g^{mb} - g^{ab}g^{ml}) + g^{al}g^{mn}(\Gamma^b{}_{lp}\Gamma^p{}_{mn} + \Gamma^b{}_{mn}\Gamma^p{}_{lp} - \Gamma^b{}_{np}\Gamma^p{}_{ml} - \Gamma^b{}_{ml}\Gamma^p{}_{np})$$
$$+ g^{bl}g^{mn}(\Gamma^a{}_{pl}\Gamma^p{}_{mn} + \Gamma^a{}_{mn}\Gamma^p{}_{pl} - \Gamma^a{}_{np}\Gamma^p{}_{ml} - \Gamma^a{}_{ml}\Gamma^p{}_{np}) + g^{ml}g^{np}(\Gamma^a{}_{nl}\Gamma^b{}_{mp} - \Gamma^a{}_{ml}\Gamma^b{}_{np})]/2 . \tag{A8b}$$

If

$$\sigma^{na}{}_b = \sqrt{-g}\, g^{ma}g^{nl}(\partial_l g_{bm} - \partial_m g_{bl})/2 , \tag{A9a}$$

we arrive at the Lorentz pseudotensor,

$$K\tau^a{}_b = \sqrt{-g}\,(\partial_b\Gamma^l{}_{pl}g^{pa} - \partial_b\Gamma^a{}_{mp}g^{mp} - \delta^a{}_b R)/2 . \tag{A9b}$$

It is clear that there is no physical criterion to select the gravitational energy-stress pseudotensor t_{ab} although all of the above give the same integral conservation law. (Recently, Bak, Cangemi, & Jackiw 1994 attempted to derive the gravitational energy-stress with a Noether current. This also results in a pseudotensor, though.) It seems that, considering the energy-stress without considering the geometry, one cannot get a valid and complete physical picture of the energy-momentum transfer (see eq. [20]).

Now experiments have proven that the current Einstein equation must be modified. These pseudotensors have become even more groundless. In general relativity, as Hilbert (1917) pointed out, there are simply no ordinary conservation laws for energy and momentum.

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