

Spacetime

The "spacetime manifold" is the smooth, continuous domain of the field. This means that spatial and time coordinates are inter-related, and that the field is a function of these coordinates. A point on the manifold is a spacetime "event", and the distance between two events on the manifold is the spacetime "interval". The interval may be "timelike", "spacelike" or "null", corresponding to whether the interval is negative, positive or zero. Practically, this means that any two events which are separated by a timelike or null interval may influence each other, since something travelling less than or at the speed of light can connect them.

The manifold can have several types of structure:

- Topology: the only measurable quantities are topological "invariants", such as the "signature" (number of timelike and spacelike dimensions), the number of holes in the manifold and whether the manifold has boundaries, or is closed. Such invariants are discrete. In General Relativity, topology change is prohibited as long as the manifold is required to have a Lorentz signature (1+3) and be globally hyperbolic.
- Affine Parameterization: the notion of parallel lines is well-defined, but distance is not. The "Christoffel Connection" is a field which allows the consistent definition of parallel lines ("geodesics") and derivatives. A "covariant" quantity is one which is consistently defined everywhere (when "parallel-transported") on the manifold. One such quantity is the "curvature", which measures the change a vector undergoes when parallel transported around a closed path. When such a change is zero everywhere on the manifold, the manifold is said to be "flat".
- Metric Parameterization: the metric is a field which defines the spacetime interval, so that distance is a measurable quantity. By requiring the angle between two spacetime vectors to remain unchanged when they are parallel transported, the metric is forced to be covariantly constant and unique.

Einstein's Equations are a set of partial differential equations which relate the curvature of spacetime to the sources of mass, energy, momentum and stress (the "stress-energy"). The curvature terms are constructed from geometric quantities (the curvature and the metric), but the source terms are not. The equations result from two fundamental assumptions:

- the "principle of general covariance", which implies that the metric is the only spacetime quantity which the "laws" of physics can depend on; and
- the "weak equivalence principle", which states that all freely falling bodies follow the same trajectories (geodesics) independent of their internal structure. This principle implies the equivalence of gravitational mass (as the source of gravitational attractions) and inertial mass (as the source of inertia, or resistance to acceleration). It also implies that accelerating "frames of reference" (coordinate systems) can be described by the curvature of the spacetime manifold (as opposed to "inertial frames", which have constant relative velocity and can exist in a flat manifold). Finally, the statement that freely falling bodies follow geodesics implies that the stress-energy has zero divergence.

The "strong equivalence principle" states that the spacetime manifold is everywhere Lorentzian, and implies the weak equivalence principle.

Because Einstein's Equations are generally covariant, they are invariant under general coordinate transformations. This implies that there is no preferred coordinate system in General Relativity. Hence there is not necessarily a global definition of time, which means that one cannot in general globally define positive and negative energy modes and therefore the usual momentum representation of particles. This also means that the decomposition of the metric components into dynamical variables and intrinsic time variables, necessary for both a Hamiltonian formulation of dynamics and isolation of the non-gauge

[degrees of freedom](#), is not compatible with general covariance.

Since the spacetime manifold in General Relativity can be arbitrarily curved, a consistent [local](#) definition of mass, energy, momentum and angular momentum is not possible. This results from the fact that all of these quantities are constructed from vectors, and in general, vectors may have different values when transported along different paths. Hence the only meaning one can give to these quantities arises from either restricting their relevance to an "[asymptotically flat region](#)" (whose curvature is confined to a finite region of spacetime), or defining them using "asymptotic [symmetry groups](#)" (whose symmetries are defined [globally](#) on the entire manifold). In either case, the quantity is then defined as a flux integral (flow through a surface) of the spacetime "[curl](#)" of the vector which generates the symmetry [transformation](#).

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