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TIME

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1 Time and Modern Physics

CHRISTOPHER J. ISHAM AND KONSTANTINA N. SAVVIDOU

*Time is a child playing, gambling;
for the kingdom is for the simple.*

Heraclitus

The subject of time

The subject of ‘time’ exercises a universal fascination. In no small part this is due to the genuinely interdisciplinary nature of the issues that arise. Thus questions about the nature of time occur in areas as disparate as physics, biology, psychology, philosophy, poetry (think of the work of T. S. Elliot), visual art, theology, music (for example, in the chanting of plainsong) and many more.

Some of these topics are covered in other chapters in this book, but in all cases – or, at least, in the more academic disciplines – a basic question is how the concept of time fits into the underlying metaphysical structure of the subject concerned. Thus, for us, a key issue is the role played by time in the foundations of modern physics. And, as theoretical physicists, we are particularly concerned with how the answer to this question relates to the various *mathematical* structures that are involved in the physicist’s account of time.

Let us begin by remarking that there are two quite different ways in which time has been viewed by physical scientists: these are known as the *absolute* and *relational* ideas of time. In essence, the difference comes down to whether or not we grant time (and space) an existence independent of material objects and processes. According to the absolute view of time (and space), time (and space) simply form the ‘arena’ of physics: the background structure within whose framework all of physics is necessarily phrased. On this view, material processes take place against the background of an independent ‘something’ called time (and space). Newtonian physics and the theory of special relativity are good examples of theoretical frameworks of this type.

On the other hand, the relational view denies time (and space) an existence independent of material objects and processes. On this view, time exists

only by virtue of the existence of matter and material events. Thus the concept of time is dependent in some way on the idea of matter. This view is famously associated with the names Leibniz and Mach. General relativity is arguably a theory of this type, although in its typical applications it also assumes certain absolute structures.

To a significant extent, modern physics oscillates uneasily between these two perspectives on time. One important issue is the way in which they are related. For example, what is the role of a ‘clock’ in this respect? Let us consider a wristwatch. On the one hand, it is made of matter and in that sense its temporal qualities are naturally associated with the second view. On the other hand, when we talk of a ‘good’ watch, we typically mean the extent to which it *measures* accurately the background absolute time of Newtonian physics – a concept that clearly accords with the first view of space and time.

But what about an atomic clock, which also is made of (quantum) matter? In this case we typically talk about the clock *defining* time, rather than measuring it. But what then is meant by a ‘good’ atomic clock: are some definitions of time ‘better’ than others; and how are they related to each other, and to the background time of Newtonian physics?

Clearly, one format for a chapter on ‘Time and Modern Physics’ would be to survey the different ideas of time in the classical and quantum versions of Newtonian physics, special relativity and general relativity. However, we have elected to follow a different route and to concentrate instead on two particular ways in which time arises in modern physics: as the parameter in temporal logic and as the parameter of dynamics. This will allow us to touch on many of the basic ideas concerning time, as well as to discuss some very recent ideas about time in physics.

Two roles for time

The nature of time is something that much occupied St Augustine: perhaps because, as he explains in his *Confessions*, he had such a ‘good time’ when he was a young man! Whatever the case may be, it is appropriate in our case to start with the following well-known excerpt from *The Confessions*:

What then is time? If no one asks me I know; if I want to explain it to a questioner; I do not know. But at any rate this much I dare affirm I know: that if nothing passed there would be no past time; if nothing were

approaching there would be no future time; if nothing were then there would be no present time.

The ideas implied here are as profound and relevant today as when the saint first stated them. One such is the universally acknowledged fact that 'time' is an elusive concept: in one sense we think we know exactly to what it refers, but when we try to pin it down it slips away – like a chimera, a will-o'-the-wisp. However, of more direct relevance to our present task is the several different roles for time that are implicit in Augustine's remarks. This theme is anticipated in the earlier comments of Aristotle, in his *Physics*, on the notion of time:

But when we perceive a distinct before and after, then we speak of time; for this is just what time is, the calculable measure or dimension of motion with respect to before-and-afterness. Time, then, is not movement, but that by which movement can be numerically estimated. And as motion is a continuous flux, so is time; but at any given moment time is the same everywhere, for the 'now' itself is identical in its essence, but the relations into which it enters differ in different connections, and it is the 'now' that marks off time as before and after. But this 'now' which is identical everywhere, itself retains its identity in one sense, but does not in another; for inasmuch as the point in the flux of time which it marks is changing the 'now' too differs perpetually, but inasmuch as at every moment it is performing its essential function of dividing the past and future, it retains its identity.

In a related context, in the Greek Orthodox Church – which inherited the tradition of Greek philosophy – it is believed that, together with faith, the study of nature () leads us to God. At the basis of this study lie what the Greeks called the 'categories', which are the properties without which nothing can be perceived or encompassed. According to Aristotle they are ten: substance, quantity, quality, relation, place, time, space, to have, to act, to be acted upon. Maximus the Confessor (one of the Fathers of the Orthodox Church) taught in regard to the category of time:

The beginning, the middle, and the end are features of all that can be divided in time, and it is also true to say, of all that we can perceive inside eternity. For time, because it has movement that can be measured, is specified numerically. Eternity, because together with existence it incorporates the category of time, has one dimension because it contains the origin of being (). If time and eternity are not without a beginning, even more are all that are contained in them.

(Maximus, *Theological and Philosophical Questions*)

It is evident that Maximus discriminates between time as motion that can be measured, and time as eternity, without the notion of change.

Let us now turn to the physical sciences, with the observation that a very important feature of time in physics is that the ‘way things are’ (more technically, the state of a system, see later) is specified at a given moment of time. In particular, in classical physics, at any given time t any proposition about the system is either true or false. Furthermore, such propositions can be combined using the operations of standard logic. Thus, if A and B are a pair of propositions, we can construct ‘ A and B ’, ‘ A or B ’, ‘ A implies B ’; and for any single proposition A , there is the negation ‘not A ’.

In this context, we can read into the remarks of Augustine, Aristotle and Maximus the following two roles that ‘time’ plays in physics:

1. Time appears as an ordering parameter in the sense that there is a separation of temporal experience into the ‘past’, the ‘present’, and the ‘future’. This ordering of ‘states of being’ gives rise to the concept of *temporal* logic. Thus, for example, if A_{t_1} is a proposition about the state of the system at time t_1 , and B_{t_2} is a proposition referring to the system at a later time t_2 , we can form the temporal conjunction ‘ A_{t_1} and then B_{t_2} ’. From this perspective, the ordering parameter in temporal logic is time viewed from the perspective of ‘being’ – a time must be specified in order to say ‘how things are’.
2. Another facet of time is to view it from the complementary perspective of ‘becoming’ – the idea of time arises in saying ‘how things change’. Thus time appears as the parameter of evolution that arises in the description of the dynamics of a physical system.

From the viewpoint of theoretical physics, it is important to understand the way in which these two aspects of time are represented mathematically in the relevant equations. In particular, we shall consider carefully the way in which real numbers are used in these two roles.

In the context of this chapter, we shall concentrate mainly on how these two roles manifest themselves in classical physics – especially in regard to the background, absolute time of Newtonian physics. Of course, in accordance with the developments of this century, one should really consider time in the context of special – and perhaps even general – relativity, in which case the relevant subjects are ‘causal logic’, and ‘relativistic dynamics’. For our purposes it will not be necessary to make this extension in detail. In later sections, however, we will say something about the famous ‘problem of time’

in the quantum theory of general relativity as this throws into doubt the fundamental status of all our standard ideas about time.

Time ordering and real numbers

The temporal ordering of events as ‘past’, ‘present’ and ‘future’ is not the only way in which temporal concepts have been construed in different cultures and ages. Thus, for example, the idea of ‘circular’ time arises in a variety of myths of eternal recurrence, in particular in the thought of ancient Greece, and of India (see Thapar, Chapter 2, this volume). If time is represented mathematically by a circle then it is clear that no real concept of history can be developed. For if an event lies in the future of a present one, then it also lies in its past.

This circular view of time contrasts sharply with the modern, linear picture, which it is often argued, has its roots in Judaeo-Christian theology. This world-view is fundamentally historical, the key events being strung out between the creation of the world and its final apocalyptic consummation. In Christianity, this linear ordering of events is additionally *centred* on the birth of Christ: every occurrence is either ‘before Christ’ (BC) or ‘in the year of the Lord’ (anno domini, AD).

Thus, of any pair of events, labelled E and F , Judaeo-Christian sensibility asserts that F lies in the future of E , or E lies in the future of F , or E and F are contemporaneous. This explains why the real numbers (which can be construed as distances along a straight line, measured from an arbitrarily fixed zero point) are a natural mathematical model for time, since for any pair of real numbers a and b it is true that $a < b$, or $b < a$, or $a = b$.

To understand more fully the role of the real numbers in the representation of time – both in regard to temporal logic and to dynamics – it is necessary to look more closely at the type of mathematical structures that are used in theoretical physics, in particular the idea of the space of states of a system. We will concentrate mainly on the situation in classical physics, and defer discussion of quantum theory until later sections.

Classical physics: the logic of propositions and the space of states

Consider an arbitrary physical object, or physical system, at some moment of time. In classical physics, a key feature of such a system is that it is completely defined by specifying all its properties at that moment – what we have referred to above as the time of ‘being’. Such a defining list of properties is called a *state* of the system. Of course, although at any given moment of time a physical system is in one, and only one, physical state (this is implicit in the concept of ‘state’), at different moments of time the system can (and generally, will) be in different states. The set of all possible states of a system is called the *space* of states, or *state space*.

To illustrate these ideas, consider the following example. Imagine a point particle (a theoretical entity of no spatial extension but with a fixed mass) moving in one dimension (i.e. along a line) under the influence of forces of a Newtonian type. It can be shown that such a system is completely defined by two properties: the position (denoted by x) and the momentum (mass velocity; denoted by p) of the system. In other words, the state of such a system is completely determined by specifying the values of the position x and momentum p of the particle; thus the space of states of the system is a two-dimensional space with coordinates x and p , as shown in Figure 1.

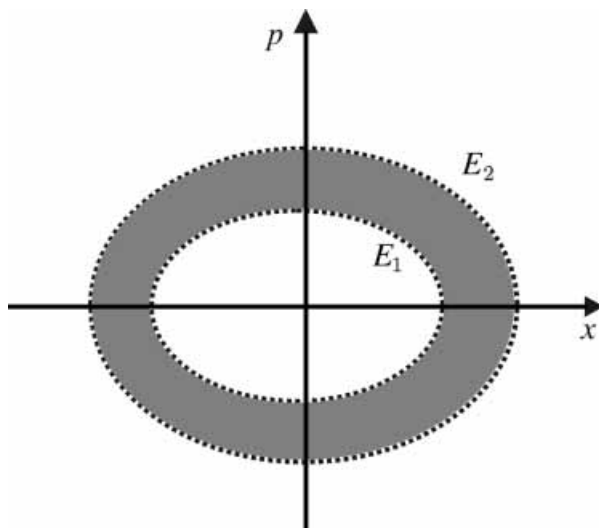


FIGURE 1. A classical statespace.

Of course, the point particle will have other physical properties besides position and momentum; for example, it will have a certain energy (denoted by E). However, the energy of the point particle is determined by its state, i.e. by the values of the position and momentum: mathematically – in the particular example in Figure 1 – we have

$$E(x, p) = \frac{p^2}{2m} + kx^2, \quad (1)$$

where m denotes the mass of the particle, and k is some positive constant. From this definition of the energy of the point particle, it follows immediately that different states can give rise to the same value of the energy. This prompts the following question: what is the set of all states (x, p) for which the proposition ‘the energy of the system has the value E_1 ’ is true? This set of states is represented by the inner ellipse in Figure 1. Similarly, the outer ellipse represents the set of states for which the proposition ‘the energy of the system has the value E_2 (with $E_1 < E_2$)’ is true. The proposition ‘the energy of the system lies between E_1 and E_2 ’ is represented by the shaded subset between the two ellipses. In fact, *every* proposition about the point particle can be represented by a set of states, namely the set of states for which the proposition is true.

This idea can be generalised to *all* classical systems: if we denote the state space of such a system by S , then *every* proposition P about the system can be represented by an associated subset S_P of S – namely the set of all states for which the proposition is true. Conversely, every subset of S represents a proposition: more precisely, every such subset represents *many* propositions, each of which asserts that the value of a certain physical quantity lies in a certain range.

It is easy to see how the logical calculus of propositions is represented in this picture. For suppose that P and Q are a pair of propositions, represented by the subsets S_P and S_Q , respectively, and consider the proposition ‘ P and Q ’. This is true if, and only if, both P and Q are true; and hence the subset of states representing this logical conjunction are those states that lie in both S_P and S_Q , i.e. the set-theoretic intersection $S_P \cap S_Q$. Thus ‘ P and Q ’ is represented by the subset $S_P \cap S_Q$ of the state space S .

Similarly, the proposition ‘ P or Q ’ is true if either P or Q (or both) are true; and hence this logical disjunction is represented by those states that lie

in S_p plus those states that lie in S_Q , i.e. the set-theoretic union $S_p \cup S_Q$ of the two sets. Finally, the logical negation ‘not P ’ is represented by all those points in S that do *not* lie in S_p , i.e. the set-theoretic complement S/S_p .

In this way, a fundamental equivalence is established between the logical calculus of the propositions about a physical system, and the Boolean algebra of subsets of the associated space of states.

Dynamics and the representation of time by real numbers

We can now use the idea of the space of states S to discuss the way in which dynamical evolution is handled in physics. The key idea here – at least, in classical Newtonian physics – is that the state of a system changes in a deterministic way with respect to the background Newtonian time t . What this means is that if s_{t_1} is the state of the system at some time t_1 , then the state s_{t_2} at any later (or earlier) time t_2 is uniquely determined by the forces acting on the system.

The precise way in which s_{t_1} changes into other states as time evolves is given by Newton’s famous second law of motion:

$$F = m \ a, \tag{2}$$

where F is the force acting on the system, m is the system’s mass and a is the acceleration. For our point particle moving in one dimension, this equation is equivalent to the following set of so-called (first-order, ordinary) *differential equations*:

$$m \frac{dx}{dt} = p \tag{3}$$

$$\frac{dp}{dt} = F. \tag{4}$$

Thus we see how the real numbers arise again as a model for time: namely as the independent parameter in the theory of ordinary differential equations. Hence, ultimately, this use of the real numbers depends on their role in *differential calculus*. This role is related, but not equivalent, to their appearance as the ordering parameter in temporal logic. Indeed, one of the main points we wish to make is that for the future development of theoretical physics in, say, quantum gravity, it may be profitable to distinguish between

these two roles more sharply than is usually the case. We shall return to these issues in later sections.

Temporal logic in this approach

Let us consider now how the temporal-logic proposition ‘ P at time t_1 and then Q at time t_2 ’ might be represented in this formalism, where (say) $t_1 < t_2$.

In our previous discussions, we saw that the proposition ‘ P and Q ’ could be represented by the subset $S_P \cap S_Q$ which comprises all those states for which P is true *and* Q is true. But notice that in the case of the proposition ‘ P at time t_1 *and then* Q at time t_2 ’ we must proceed more carefully, since we are comparing states at different moments of time.

We can suppose that, as in the discussion above, the propositions about the system are represented by subsets of the state space S . Furthermore, if we think of the states in S as being the states of the system at time t_1 (thus we chose t_1 as an initial ‘reference’ time), then the proposition ‘ P at time t_1 ’ corresponds to the subset S_P . The main step now is to find the mathematical representation of the proposition ‘ Q at time t_2 ’ as a subset of the set S of states at time t_1 .

In this context, we recall from the discussion above that states evolve deterministically, which means that every state at t_1 has a unique successor state at t_2 ; conversely, every state at t_2 has a unique predecessor state at t_1 . In particular, each state s at time t_2 that makes Q true (i.e. s lies in S_Q) has its own unique predecessor state at t_1 . We will denote by $(S_Q)_{\text{pred}}$ the subset of S of all such predecessors of states that lie in S_Q ; clearly, this is the subset of S that represents the proposition ‘ Q at time t_2 ’.

Of these predecessor states, some will lie in S_P , and others will not. Thus the subset that represents our proposition ‘ P at time t_1 *and then* Q at time t_2 ’ consists of those predecessor states in $(S_Q)_{\text{pred}}$ that also lie in S_P ; namely, the set-theoretic intersection

$$S_P \cap (S_Q)_{\text{pred}} \tag{5}$$

This is a perfectly workable definition and it is implicit in the usual treatment of classical mechanics, but it has the feature that the set-theoretic representation of the temporal-logic proposition depends explicitly on the detailed *dynamics* of the system (via the evolution of subsets of S into other

subsets). From our perspective this is undesirable, since it mixes completely the two uses of time: what we have called the time of being and the time of becoming. And, anyway, it is anomalous in the sense that the physical *meaning* of the proposition ‘ P at time t_1 and then Q at time t_2 ’ is independent of the dynamics, and hence it is natural to require that there be a mathematical representation of the proposition with the same property. This can indeed be done, as we shall now see.

A history version of classical physics

It may seem surprising, but it seems to have been only very recently that the possibility of finding a dynamics-independent representation of temporal logic was discussed. In fact, the question arose first in the context of quantum theory; only after the issue was discussed there was it realised that there is an analogous construction in the classical case (as discussed by K.N.S. in 1999).

The mathematical framework involved in the case of classical physics is illustrated in Figure 2. The key idea is to start *ab initio* with two time variables – a time of ‘being’ and a time of ‘becoming’ – and then to associate with each time of being t a *separate* copy S_t of the classical state space, which is to be thought of as representing the states of the system at that time t .

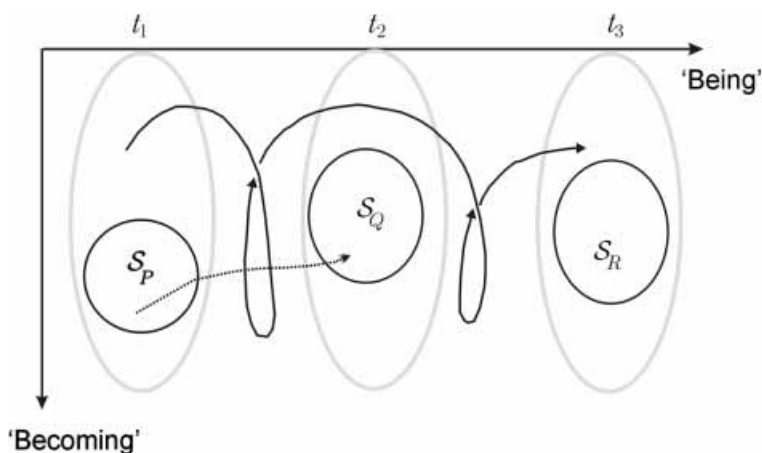


FIGURE 2. History formalism for classical physics.

A natural mathematical quantity associated with this construction is a *path*: this is a function that, for each time t , assigns a state (t) that lies in the state space S_t . From a physical perspective, each such path describes a possible *history* of the system: i.e. (t) describes ‘how things are’ at the ‘time of being’ t .

A proposition like ‘ P at time t_1 and then Q at time t_2 ’ is then represented by the set of all *paths* with the property that at the time t_1 , the state (t_1) lies in the subset S_P of S_{t_1} , and at the time t_2 , the state (t_2) lies in the subset S_Q of S_{t_2} . This representation of temporal propositions by subsets of paths is clearly independent of dynamics, and it is straightforward to check that it reproduces the expected logic of temporal propositions.

The idea of dynamics arises in an interesting way in this framework. From a mathematical perspective, there is a dynamical evolution associated with *each* copy S_t of the state space: thus the time of temporal logic and the time of dynamics are now separate from each other.

This is reflected in the fact that there are *two* types of time transformation in this formalism: one that transforms the ‘external’ time indices on temporal propositions (the time of ‘being’) and one that is associated with the internal evolution (the time of ‘becoming’) in each S_t for every value t of the external time label t . However, it must be emphasised that we are *not* claiming that ‘physical’ time should be represented by a two-dimensional space (in addition to the three dimensions of physical space; so that space-time would be five-dimensional): in fact, for any given physical system the two types of time transformation are locked together, and the actual history of the system is a single path, as represented by the spiral in Figure 2.

It came as a great surprise to us to discover that the central idea behind the two types of time transformation had to some extent been anticipated by the artist John Latham. In a series of works known generically as ‘time-base rollers’ he has presented a graphic representation of these ideas: an example is the picture in Figure 3. The axis going horizontally along the length of the roller (what Latham refers to as the ‘time-base’) is the analogue of our ‘time of being’, while the dynamical evolution associated with each such time is represented by the vertical motion of the blind as it is unrolled.

From a philosophical perspective, certain ideas of process philosophy underpin Latham’s general metaphysical views, and it is interesting to ask

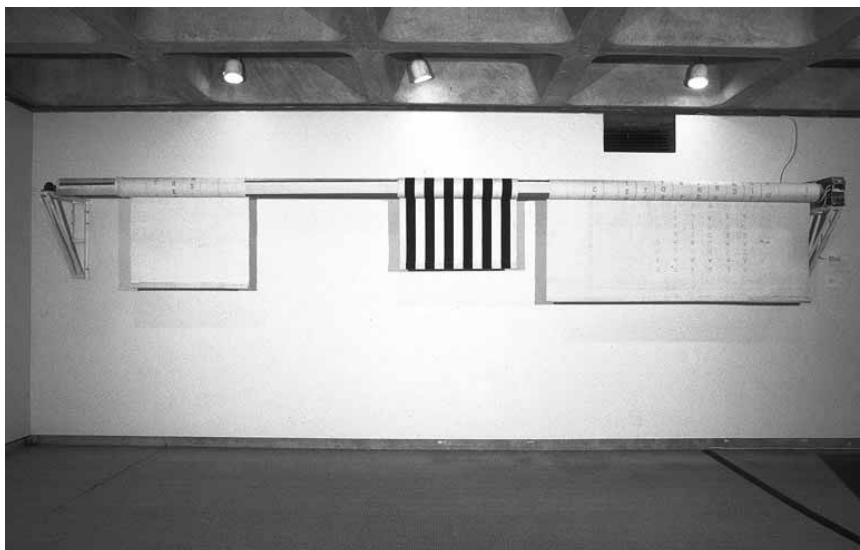


FIGURE 3. One of John Latham's time-base roller constructions.

how process ideas might be reflected in the theoretical physicist's representation of time.

The mathematics of process

We shall now address once more the important question of how the 'time of being' and the 'time of becoming' are related to each other in theoretical physics. In particular, we want to consider how one might mathematically implement the vision of a process philosopher such as A. N. Whitehead in which the notion of the time of 'becoming' is structurally independent of the time of being, whereas the converse is normally the case (see below). Indeed, Whitehead emphasises that, from a purely empirical perspective, the notion of 'event' – and the relation between events – is more fundamental, and our normal concepts of space and time should be understood as idealised mathematical constructs from the empirical data pertaining to events.

This is reflected in his concept of the *discernible*, which is to be viewed as everything that happens in some (time) *duration*. The concept of a 'duration' is irreducible in the sense that it is *not* to be construed as a collection of 'moments of time'. A key property of durations is their ability to contain one

another, and it is this type of ordering property that Whitehead considers to be fundamental. On the other hand, there is the notion of *instantaneousness*, which is an idealised, logical concept of all nature at an instant, where an instant is conceived of as deprived of all temporal extension; from this concept also arises the idea of a ‘moment’ of time.

Thus we are interested in finding a mathematical representation of ‘durations’ that does not depend on the idea of a ‘moment’ of time (which is represented normally by a real number). To this end, let us consider again the way in which the time of becoming is represented mathematically in standard classical physics. This involves considering the mathematical structure of the dynamical laws of classical physics.

The dynamical equation of elementary classical physics is Newton’s second law of motion, which, as we have seen before, can be written as the coupled differential equations

$$m \frac{dx}{dt} = p \tag{6}$$

$$\frac{dp}{dt} = F. \tag{7}$$

For our purposes it is important to note that the time derivative dp/dt in equation (7) is to be interpreted mathematically as the limit

$$\text{Limit} \frac{p(t) - p(t)}{t - t} \tag{8}$$

as t gets closer and closer to t . A similar remark applies to the time derivative dx/dt in equation (6).

Now, as we emphasised earlier, the ‘time of becoming’ variable arises mathematically via the theory of differential equations and differential calculus, as in equations (6) and (7). On the other hand, the quantities t and t in equation (8) are points of the ‘time of being’, since they label the specific times at which the momentum variable p has a value. This is the sense in which the concept of ‘becoming’ is structurally dependent on the concept of ‘being’.

From a mathematical perspective, the reason for this dependence is that there are no genuine infinitely small numbers (so-called ‘infinitesimals’) in standard mathematics, and hence the symbol ‘ dt ’ has to be interpreted in a

limiting sense, as in equation (8). However, there is a subject known as synthetic differential geometry in which genuine infinitesimals *do* exist. This suggests that dynamics could be formulated in a new way in which the concepts of ‘becoming’ and ‘change’ are independent of any underlying points of temporal being; specifically, one could try to identify the infinitesimals as the mathematical analogues of Whitehead’s ‘durations’.

By these means one would gain an implementation in mathematical physics of certain ideas of process philosophy. However, in doing so it is necessary to introduce a non-standard model of the real numbers, and – it transpires – an underlying logical structure that is *intuitionistic*. In standard logic we have the principle of excluded middle, which asserts that the disjunction of any statement with its negation is always true. In intuitionistic logic, contrary to standard logic, the principle of excluded middle no longer holds, i.e. propositions of the form ‘*P* or (not *P*)’ are not necessarily valid. We note *en passant* that the inapplicability of the principle of excluded middle is a characteristic feature of the so-called *constructive* approach to mathematics in general.

Time in quantum theory

There are analogues in quantum theory of the various aspects of time in classical physics that we have discussed above, albeit complicated by the special role of the idea of ‘measurement’ in the standard interpretation of the theory.

Quantum theory is of fundamental importance in describing the world at atomic and subatomic scales. However, it differs from classical physics in one very important respect: in classical physics, in any specific state, a proposition such as ‘the particle is at position x ’ is either true or false. On the other hand, in quantum physics, the best that can be done is to talk about ‘the probability that, *if* a measurement of position is made, the particle will be found at x ’, and this probability is assigned a numerical value between 0 and 1.

This strict instrumentalism that lies at the heart of the standard interpretation of quantum theory, means that the concept of ‘being’ is replaced by that of ‘being measured’; which leads one to wonder what happens to the distinction made earlier between the time of temporal logic (‘being’) and the time of dynamics (‘becoming’)?

In regard to dynamics, the answer is that – provided no measurements are made – quantum states, and their associated probabilities, evolve deterministically with respect to a background Newtonian time: so in this respect classical and quantum theory are alike. In particular, the dynamical evolution of the mathematical object (a vector in a vector space) that represents a quantum state is described by a differential equation (the Schrödinger equation), and the remarks made earlier about the implications of this for the mathematical model of time apply here too.

However, in the standard interpretation of quantum theory there is another type of time evolution that can occur. This arises if a measurement of some physical quantity is made: the quantum state is then deemed to change instantly into a different one that reflects the *actual* result obtained from among the set of those that are probabilistically possible according to the theory. Many physicists feel this to be one of the most unsatisfactory features of standard quantum theory, and there have been many approaches to the task of developing a theory in which these sudden apparent ‘collapses’ of the state vector can be derived in some way; for example, from the deterministic evolution of the state vector of the system that includes the measuring device, now regarded as a quantum entity in its own right. One such scheme – the *consistent histories* approach to quantum theory – bears directly on our topic of the two facets of time.

In regard to temporal logic, the standard quantum formalism certainly allows the assignment of a probability to getting a particular sequence of results of measurements of various observables made at a sequence of times, but this depends explicitly on the details of the dynamics of the system, and thus we once again have a situation in which the time of temporal logic is mixed with the time of dynamics.

What is needed to separate these two aspects of time is a quantum analogue of the history formalism discussed above for the classical case. But to do that it would be necessary to work with a version of quantum theory that deals with sequences of *values* of quantities, not results of the *measurements* of quantities – something that is not possible in standard quantum theory. However, in recent years a new approach to quantum theory has been developed, known as the *consistent histories theory*, which is aimed at this precise point. The penalty extracted by the quantum realm for being allowed to talk about sequences of values (rather than results of

measurements) is that probabilities can be assigned only to a limited class of propositions.

This theory has been developed by ourselves and co-workers in such a way as to place emphasis on the idea of quantum temporal logic. In this form it is relatively easy to implement the idea of two times and the associated two types of time transformation: indeed, as remarked earlier, it was in the context of this theory that the idea of the two types of time variable first arose – only afterwards was it appreciated that there is an analogous structure in classical physics.

It is tempting perhaps to connect the two types of time evolution in standard quantum theory (deterministic dynamics, and the collapse of a state vector when a measurement is made) with the two types of time transformation that arise in the history theory. Indeed, one of us (K.N.S.) has conjectured the existence of a close correspondence between the collapse of the state vector and changes in the time of 'being'.

Spacetime, gravity and a variety of times

The central idea of special relativity is that the three-dimensional space and one-dimensional time of Newtonian physics are combined together to give a single, four-dimensional structure known as *spacetime*. This is a radical step but, nevertheless, the role thereafter of spacetime is not dissimilar to that played in Newtonian physics by the separate concepts of space and time. In particular, the geometry of the spacetime structure (which determines things like the distance between two points in spacetime) is fixed and forms the background within which the dynamical equations of physics are framed. (It is not just fixed, but also relatively simple: we talk of the spacetime in special relativity being 'flat'.)

The situation in general relativity is quite different. The geometry of spacetime is no longer fixed, but instead depends on the energy and matter present in the universe. This dependence is captured by Einstein's famous field equations, which describe the precise manner in which the 'geometric field' (which corresponds physically to the gravitational field) depends on the distribution of matter and energy in the universe. The result of this dependence is that spacetime is no longer 'flat', but instead becomes 'curved'.

The idea that spacetime is curved has a radical effect on our concept of time. Einstein's field equations have many different solutions, leading to many different spacetimes. Many of these have the special property that the four-dimensional spacetime can be regarded as a 'stack' of curved three-dimensional spaces, each of which is given a unique label that can be thought of as the 'moment' of time corresponding to that three-dimensional space. In this sense, a four-dimensional spacetime can be thought of as a history of three-dimensional space. The events occurring in the four-dimensional spacetime can be time-ordered using the labels of the three-dimensional spaces in which they are located. In this respect, the picture looks similar to that of Newtonian physics.

However, a key feature of general relativity is that the manner in which a four-dimensional spacetime is divided into a stack of three-dimensional spaces is very non-unique. In fact, almost any way of dividing the spacetime can be chosen provided only that the ensuing stack of three-dimensional spaces has the property that no light beam can travel across any single one of them. Since nothing travels faster than light, the key idea here is that any entity must move from one element of the stack to the other as 'time' increases.

All these different ways of carving up four-dimensional spacetime into three-dimensional spaces result in different time orderings of events. Whether or not any particular carving-up is admissible, and yields a well-defined time, depends on the geometry of spacetime and hence on the distribution of matter in the universe. This picture is far removed from that of the fixed, universal time of Newtonian physics.

The problem of time in quantum gravity

The central feature of time in quantum gravity arises from the fact that, as emphasised above, in general relativity what constitutes an admissible way of labelling events as occurring at different times depends on the geometry of spacetime. However, in a theory of quantum gravity (which combines the theory of general relativity with quantum theory) we expect this geometry to have the same type of quantum features as we discussed above: in particular, the geometry will not have a definite value, but will be determined only probabilistically. But if the spacetime geometry had no definite value, then presumably neither would the set of allowed ways of introducing time:

in particular, there would be no single choice that can serve as a uniform way of labelling events as occurring at different times for all the spacetime geometries that could appear with non-zero probability.

A detailed study of what is known as the ‘canonical’ quantisation programme confirms these expectations: there is indeed something like a probabilistic distribution of three-dimensional geometries, but with no time label at all! Making sense of this peculiar situation is known as the *problem of time* in quantum gravity.

Many different ways have been suggested for recovering the notion of time in quantum gravity. One possibility is that there is some universal choice for time that stands outside the internal structure of general relativity. This suggestion tends to be particularly popular with process philosophers and theologians who have an interest in theoretical physics. However, since it seems to violate the spirit – if not the law – of general relativity it does not appeal to the majority of those who work in quantum gravity.

In practice, most research in canonical quantum gravity has appealed to the notion of ‘internal’ time in which part of the gravitational field is used as a local clock to specify the time at which the remaining parts of the gravitational field have a probabilistic distribution. This relational view of time has attractive features but it also raises some difficult questions. In particular: (i) can a choice for internal time be found such that the ensuing dynamical equations for the rest of the field are of the type encountered elsewhere in quantum theory; and (ii) if this is the case, can the predictions associated with different such choices (assuming there are such) be related in a physically meaningful way?

Unfortunately, the prognosis for answering ‘Yes’ (or, at least, nearly ‘Yes’) to these questions is not good. The evidence suggests that the dynamical equations agree only approximately with standard ones: a situation that has led to suggestions that what we normally call ‘time’ will emerge from the formalism only in a coarse-grained way (analogous, perhaps, to the ideas of temperature or pressure in the theory of a gas). This throws into severe doubt the validity of the probabilistic structure as a fundamental ingredient of the theory, and hence the viability of the quantum formalism as a whole when applied in the context of quantum gravity. Also, model calculations suggest that different choices of internal time would lead to different families of probability distributions, and in the absence of any background spacetime

reference system it is difficult to see how the different results could be compared physically.

Under these circumstances, we may well wonder what happens to our two different concepts of time: the time of temporal logic ('being'), and the time of dynamics ('becoming')! Our suspicion is that, in order to answer such questions, it will be necessary to introduce radical revisions in the basic ideas of quantum theory itself.

Zervan and the creation of time

One can often find silver linings in life, and in the present context the obscurities in the concept of time in quantum gravity allow the possibility of developing theories of *cosmogenesis* that deal with the origin of the universe itself, including the quixotic concept of the 'origin of time'. The most famous quantum cosmology theory of this type is that due to Jim Hartle and Stephen Hawking.

The problem of the 'origin of time' has preoccupied certain earlier civilisations. For example, the ancient Persian religion of Zoroaster conceived of a primordial god, Zervan (see Figure 4), who existed by, and in, himself –

– before the origin of time. Feeling alone, he wished to conceive in order to have a companion, and being alone, but also being a God, he was obliged to do this by offering sacrifices to himself that he might have a son. But, during this process, he began to doubt if his sacrifices would work, and from this doubt there eventually came to be another son, Ahriman – the principle of negation – who was born before the good son, Ohrmazd, whom Zervan desired.

Being fair minded, Zervan had to allow his first-born son Ahriman to rule for a while, but he also decreed that he should be overcome by Ohrmazd, the power of good. And to allow this battle to take place, Zervan created the physical world, and what we call 'time' as the background against which this cosmic struggle can occur. Note that a common gnostic theme is that evil – which is also often associated in some way with the material world – arose as a hypostasis of the doubt of a high spiritual being.

It is interesting to note the ascending spiral in the picture of Zervan. Bearing in mind our earlier remarks about spirals and dynamics, it is clear that the ancient Persians also knew about the two times!

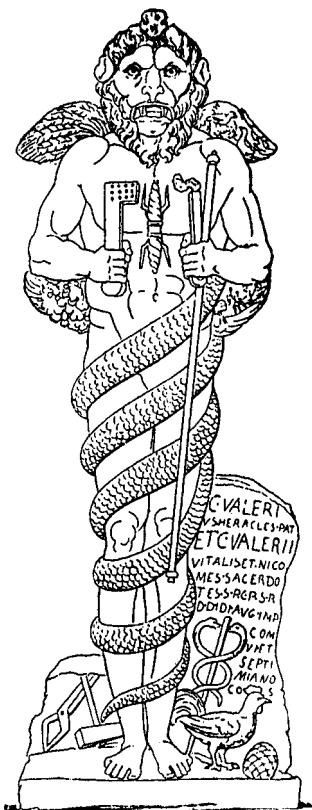


FIGURE 4. The god Zervan.

Conclusions

We have argued that there are two main uses for the concept of time in physics: (i) as the parameter in temporal logic, labelling the points of 'being'; and (ii) as the parameter in the equations of dynamics, where it refers to the notion of 'becoming'.

The full implementation of the ideas of temporal logic in a dynamics-independent way requires a *history* formulation of physics, and we have illustrated briefly how this arises in the classical case.

We have emphasised that a key question for theoretical physics concerns the appropriate mathematical structure that is used to represent the idea of time. In particular, the real numbers appear as: (i) the ordering parameter

that labels the points of being; and (ii) the parameter that pertains to becoming, via dynamics with its use of differential calculus and the theory of differential equations. The standard way of defining differentiation – via a limiting procedure – necessarily mixes up the two ideas of time. However, there are other models of real numbers – in particular, in synthetic differential geometry – which allow for genuine infinitesimals, and which hence offer a way of separating mathematically the two concepts of time.

Then we discussed briefly the deep question of time in quantum gravity, and the possibility that what we normally call ‘time’ is not a fundamental concept but rather one that arises only in some sort of coarse-grained way. Ideas of this sort give rise to the possibility of constructing quantum theories of the origin of the universe and, in particular, the ‘beginning’ of time.

Finally we note that in the course of this chapter we have introduced ideas from physics, mathematics, philosophy, theology, visual art and Persian gnosticism. If nothing else, this should suffice to demonstrate the remark made in the beginning of this chapter about the interdisciplinary nature of interest in time!

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